A structure-preserving staggered semi-implicit scheme for continuum mechanics

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In this talk, we present a new class of structure-preserving semi-implicit schemes for the unified first order hyperbolic model of Newtonian continuum mechanics proposed by Godunov, Peshkov and Romenski (GPR). The GPR model is a **geometric approach** to continuum mechanics, which is able to describe the behavior of moving elasto-plastic **solids** as well as viscous and inviscid **fluids** within one and the same governing PDE system. This is achieved via appropriate relaxation source terms in the evolution equations for the distortion field and the thermal impulse. In previous work it has already been shown that the GPR model reduces to the compressible Navier-Stokes equations in the stiff relaxation limit when the relaxation times tend to zero.
The governing PDE system belongs to the class of symmetric hyperbolic and thermodynamically compatible systems (SHTC), which have been studied for the first time by Godunov in 1961 and later in a series of papers by Godunov & Romenski. An important feature of the proposed model is that the propagation speeds of all physical processes, including dissipative processes, are *finite*.

In the absence of source terms, the homogeneous part of the GPR model is endowed with some natural **involutions**, namely the distortion field **A** and the thermal impulse **J** need to remain **curl-free**. In this talk we present a new **structure-preserving** scheme that is able to preserve the curl-free property of both fields **exactly** also on the discrete level. This is achieved via the definition of appropriate and compatible discrete gradient and curl operators on a judiciously chosen staggered grid. Furthermore, the pressure terms are discretized implicitly in order to capture the low Mach number limit of the equations properly, while all other terms are discretized explicitly. In this manner, the resulting pressure system is symmetric and positive definite and can be solved with efficient iterative solvers like the conjugate gradient method. Last but not least, the new staggered semi-implicit scheme is also able to reproduce the **stiff relaxation limit** of the governing PDE system properly, recovering an appropriate discretization of the compressible Navier-Stokes equations.
To the best of our knowledge, this is the first pressure-based semi-implicit scheme for nonlinear continuum mechanics that is able to preserve all involutions and asymptotic limits of the original governing PDE system also on the discrete level.
Computational results for several test cases are presented in order to illustrate the performance of the new scheme.

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