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Modelling Turbulent Deflagrations in Type Ia Supernovae

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We present an overview of the current state of multidimensional modelling of type Ia supernovae and an example for surprising consequences of impoving the physics of the model. In this case, an improved handling of subgrid scale turbulence gives rise to lower burning rates and a decreased global energy release. While this result is too preliminary to be interpreted quantitatively, it shows that much work remains to be done before the turbulent deflagration model can be declared to be understood and/or insufficient to explain normal type Ia supernovae.

1. INTRODUCTION

Type Ia supernovae (SNe Ia) have received a lot of attention recently in their roles as cosmological distance indicators and prime witnesses of the accelerated expansion of the universe (see [1,2] for reviews). Apart from cosmology, there is an urgent need for reliable SN Ia models in the context of nucleosynthesis of heavy elements and galactic chemical evolution. Yet despite more than three decades of research we are still debating some fundamental aspects of the physics of the stellar explosions that give rise to these bright and powerful events.

As is widely known, the most successful model for a SN Ia is the thermonuclear explosion of a CO-White Dwarf driven to criticallity by accretion in a binary system until it reaches the Chandrasekhar mass (e.g., [3]). "Successful" means that in principle, as demonstrated by spherically symmetric explosion models, the right amounts of ^{56}Ni and intermediate mass elements can be produced (with adequate velocities) to explain the light curves and spectra of typical SNe Ia. Unfortunately, in these models much of the essential explosion physics is hidden in the parametrization of the thermonuclear burning speed. In the case of the initial subsonic deflagration phase, the speed is basically a free parameter whereas for the proposed secondary phase of a delayed detonation [4,5], the speed is fixed by hydrodynamics but the onset of the detonation is dialed in by hand. Furthermore, in the latter case the non-spherical geometry of an off-center detonation cannot be captured in 1D-simulations.

Much progress has been made recently in the field of multidimensional (2 and 3D) simulations of SN Ia explosions [6–8]. Parameter studies with 3D simulations of the turbulent

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deflagration phase have been presented in [9] and work is beginning now to model delayed detonations in multiple dimensions [10,11]. However, in spite of similar results the interpretation of the current state of affairs differs somewhat among various groups. While the generally low ^{56}Ni-masses and ejecta velocities of the 3D deflagration models as well as the residual unburnt C+O near the stellar core has been interpreted as clear evidence for delayed detonations in normal SNe Ia by [10], we argue here that this assertion is premature based on the following arguments: i) the trends in highly resolved 3D deflagration models point toward a resolution of the core-CO problem as a more realistic representation of turbulent entrainment is reached by increasing the effective numerical Reynolds number (limited to $\mathcal{O}(10^3)$ in current models); ii) delayed detonations require spontaneous deflagration-detonation-transitions (DDT) whose occurence in unbounded flows is not well understood [12] and must therefore be initiated by hand in current models; and iii) even if they occur, it is unclear whether they can really cure all the shortcomings of pure deflagration models [11].

In the remainder of this article, we will sketch some new developments regarding the modelling techniques for turbulent thermonuclear flames. This work is still in progress and is meant to serve as an example of how much work still needs to be done to conclude that we really understand the deflagration phase.

2. The modelling of turbulent deflagrations

In the pure deflagration model for type Ia supernovae, subsoncially propagating flame fronts consume carbon and oxygen and produce heavier elements by thermonuclear burning. From the microscopic point of view, the propagation speed s is determined by the thermal conductivity of the fuel and is typically much less than the speed of sound [13]. This process is called a deflagration. In numerical simulations of supernova explosions, however, it is impossible to explicitly resolve the burning process on the characteristic length and time scale of the thermonuclear reactions. In addition, the interaction between flame propagation and fluid turbulence folds and wrinkles the flame fronts on all length scales larger than a certain cut-off, which is called the Gibson scale (see [14] for a discussion of the relevant combustion physics in SNe Ia).

Although the Gibson scale is usually much larger than the microscopic burning scale, i. e., the flame width, not even the whole range of scales above the Gibson scale can be numerically resolved. As a consequence two approximations are made. Firstly, flame fronts are numerically represented as discontinuities. Secondly, an effective propagation speed s_{t} is introduced which accounts for the effect of turbulence on length scales smaller than the size Δ of the cells in the numerical discretisation of the hydrodynamical equations.

Representing the flame fronts as discontinuities is achieved by means of the level set method in the Prometheus code. Details about the method and implementation can be found in [15]. For the determination of the turbulent flame speed s_{t}, on the other hand, a model based on the kinetic energy associated with unresolved turbulent eddies was formulated [16]. The basic idea is that turbulent eddies of size smaller than the numerical resolution Δ will further wrinkle the physical flame front in comparison to the numerically computed front. Because the rate of energy generation by theromuclear burning grows in proportion to the surface area of the flames, this would produce too little burning in the numerical simulation. However, the numerical smoothing of the flame front can be compensated by enhancing the propagation speed. The corresponding velocity scale is given by the mean kinetic energy per unit mass of eddies smaller than Δ, which is denoted by k_{sgs} and is called the *subgrid scale turbulence energy*. Hence, the following ansatz for calculating the flame propagation speed is made:

$$s_{\mathrm{t}} = \max(s, \sqrt{2k_{\mathrm{sgs}}}). \tag{1}$$

This expression implies that s_{t} equals more or less the microscopic propagation speed s if there is no or only little turbulence. On the other hand, if $k_{\mathrm{sgs}} \gg s$, then the flame dynamics is largely dominated by turbulence and the propgation speed becomes asymptotically independent of s. The computation of k_{sgs}, in turn, entails the problem of *subgrid scale modelling.*

The kind of subgrid scale model we adopted for simulating turbulent burning in supernovae is based on a dynamical equation for the budget of turbulence energy:

$$\begin{aligned}\frac{\mathrm{D_{sgs}}}{\mathrm{D}t}k_{\mathrm{sgs}} - \frac{1}{\rho}\boldsymbol{\nabla}\cdot\left(\rho C_{\kappa}\Delta_{\mathrm{eff}}\sqrt{k_{\mathrm{sgs}}}\boldsymbol{\nabla}k_{\mathrm{sgs}}\right) = \\ C_{\nu}\Delta_{\mathrm{eff}}\sqrt{k_{\mathrm{sgs}}}|S^{*}|^{2} - (\frac{2}{3} + C_{\lambda})k_{\mathrm{sgs}}d_{\mathrm{sgs}} - C_{\epsilon}\frac{(k_{\mathrm{sgs}})^{3/2}}{\Delta_{\mathrm{eff}}}.\end{aligned} \tag{2}$$

The left hand side corresponds to the Lagrangian time derivative of k_{sgs} corrected by a diffusion term, which accounts for the subgrid scale transport of k_{sgs}. On the right hand side, there is a production term, a compression term and a dissipation term. Kinetic energy on subgrid scales is produced by non-linear turbulent interactions with the resolved velocity field. The rate of production is proportional to the turbulent viscosity $\nu_{\mathrm{sgs}} = C_{\nu}\Delta_{\mathrm{eff}}\sqrt{k_{\mathrm{sgs}}}$ and the rate of strain $|S^{*}|^{2} = 2S_{ik}S_{ik} - \frac{2}{3}d^{2}$, where $S_{ik} = \frac{1}{2}(v_{i,k} + v_{k,i})$ is the symmetrized velocity derivative and $d = v_{i,i}$ the divergence. The rate of dissipation is given by the dimensional expression k_{sgs}/τ_{ϵ}, with the dissipation time scale $\tau_{\epsilon} = \Delta_{\mathrm{eff}}/(C_{\epsilon}\sqrt{k_{sgs}})$. In order to solve the above equation for k_{sgs}, it is necessary to compute the so-called closure parameters, in particular, C_{ν}, C_{ϵ}, C_{κ} and C_{λ}. In the following, we will concentrate on the parameter of production, C_{ν}.

The simplest possibility is to assume a constant value of C_{ν}. For statistically stationary and isotropic turbulence $C_{\nu} \approx 0.5$ is actually a fair approximation. However, the Rayleigh-Taylor driven turbulence in a supernova explosion is actually transient and rather inhomogenous. For this reason, corrections of C_{ν} corrsponding to the local properties of the flow are required for a sound subgrid scale model. So far, a simple ad hoc rule (a so-called wall proximity function) has been used in our simulations of type Ia supernovae [16]. A presumbly much better approach makes use of the self-similarity properties of turbulence in the limit of small length scales. The underlying assumption is that, regardless of the large-scale structure of a flow, turbulent eddies become asymptotically scale-invariant towards smaller length scales. This is known as inertial subrange of length scales. If the resolution in a numerical simulation is sufficient, then the smallest resolved as well as the subgrid scales will be within the inertial subrange and exhibit scale invariance. It can be shown that the local value of the parameter C_{ν} can be estimated from structural properties of the resolved flow in this case [17]. Because C_{ν} then varies in space and time, we speak of a localised subgrid scale model.

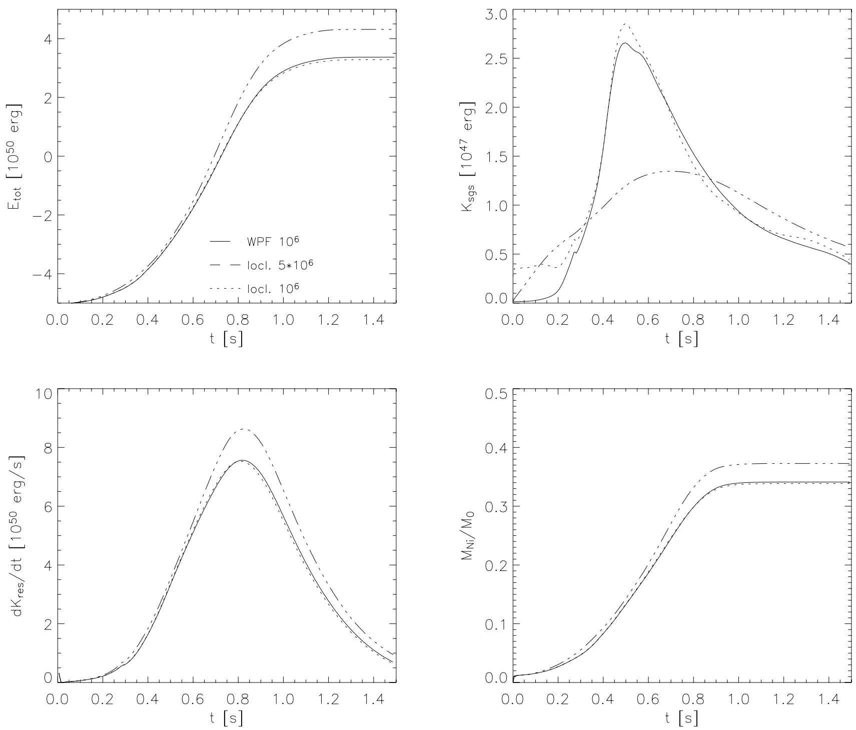

Figure 1. Total energy and mass as functions of time in simulations of thermonuclear supernova explosions using different subgrid scale models for the calculation of the turbulent burning speed.

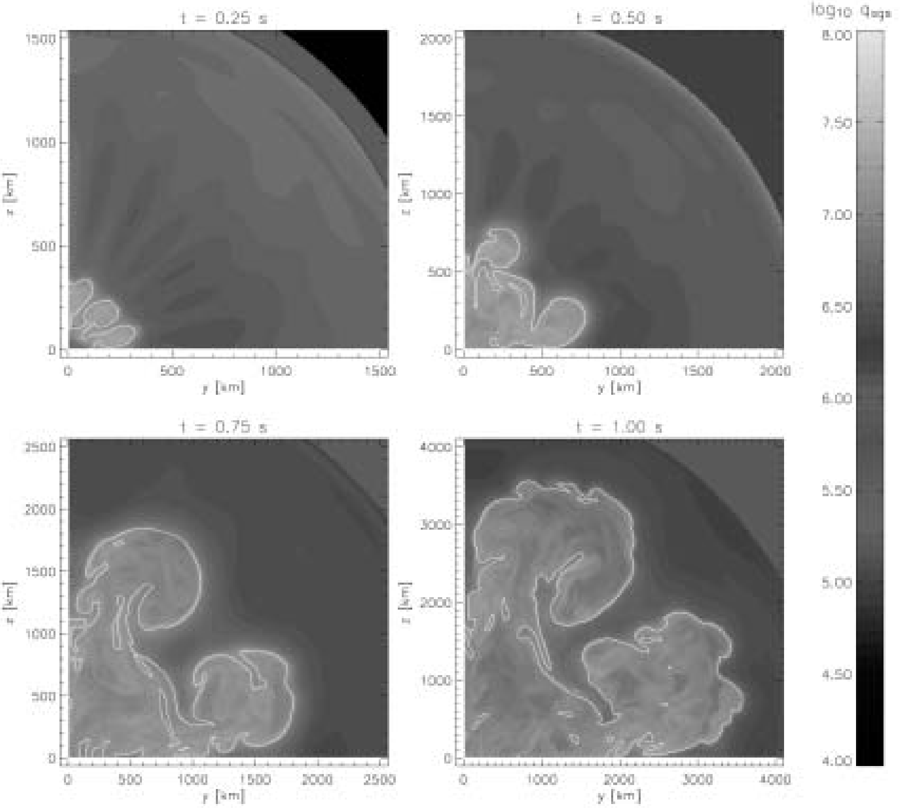

Figure 2. Contour sections of the subgrid scale turbulence velocity $q_{\rm sgs}$ for the model with wall proximity functions. The white lines correspond to the flame surface.

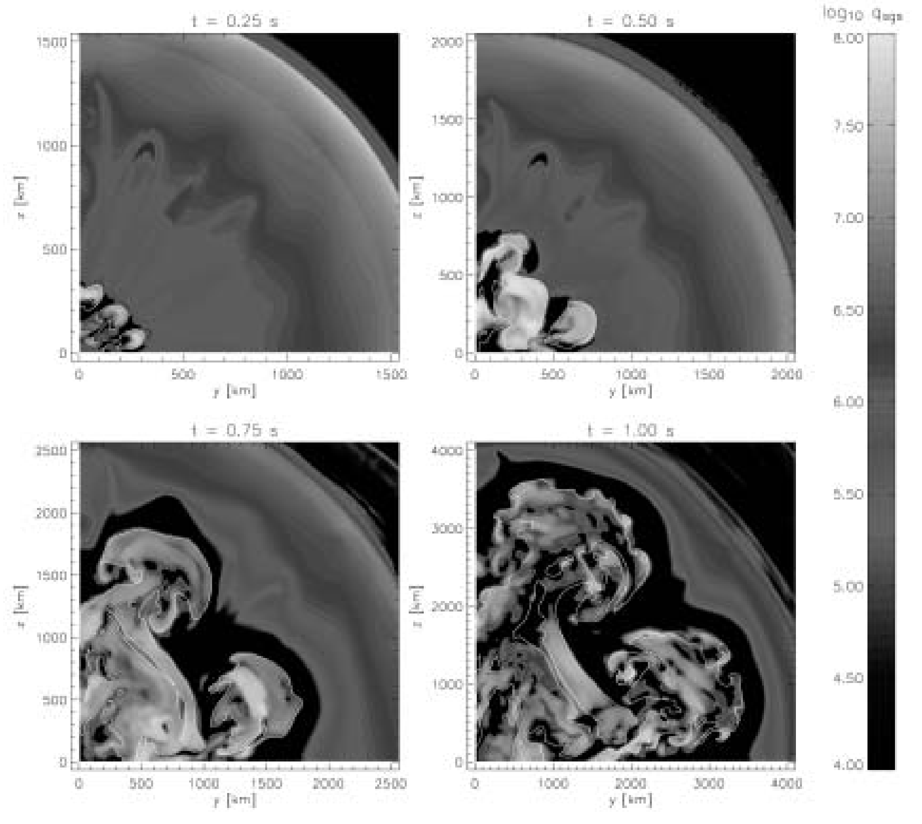

Figure 3. Contour sections of the subgrid scale turbulence velocity $q_{\rm sgs}$ for the localised model. The white lines correspond to the flame surface.

In order to compare the performance of the different subgrid scale models, we ran three-dimensional simulations of supernova explosions with a resolution of 256^3 grid cells, using axisymmetric initial conditions. The time evolution of several integrated quantities is shown in figure 1. The total energy and the subgrid scale turbulence energy are plotted in the top panels, whereas the bottom panels show the rate of kinetic energy production and the total mass of the main burning product, ^{56}Ni. The graphs labeled with WPF are obtained for the simple subgrid scale model with the wall proximity function. The other graphs apply for the localised model with two different initial values of $q_{\mathrm{sgs}} = \sqrt{2k_{\mathrm{sgs}}}$. Remarkably, the localised subgrid scale model predicts a reduction in the explosion energy. This comes rather unexpected, because previous studies based on the simulation of turbulent deflagration in a box indicated more rapid burning with the localised model. However, if one considers the distribution of turbulence energy an explanation becomes apparent. In figure 3, two dimensional contour sections of q_{sgs} are plotted for various stages of the explosion in the simulation with the localised model. Comparing this with figure 2 which shows the corresponding plots in the case of the model with the wall proximity function, one can see that the localised model predicts considerably higher intermittency of subgrid scale turbulence. Although the enhanced intermittency causes a more wrinkled flame surface, the propagation of the flames is inhibited in regions of very low subgrid scale turbulence energy. Thus, the overall energetics of the burning process is reduced. On the other hand, our treatment of subgrid scale dissipation is not fully satisfactory yet, and it is possible that an improved closure will significantly alter the trends found so far. For the time being, our major conclusion is that the global statistics of the supernova simulations appears to be sensitive to the subgrid scale model in use.

3. OUTLOOK

The results presented in the previous section show a dependence of the global energy release on the microscopic dissipation mechanism encoded in the subgrid scale model, albeit a very weak one, contrary to expectations from dimensional analysis and previous simulations of deflagrations in a box. They are very preliminary and need to be supplemented by extensive parameter and resolution studies. This work is currently in progress. However, these results demonstrate that our understanding of the turbulent deflagration phase in type Ia supernova explosions is still incomplete. This fact, together with guidance from Occam's Razor, leads us to conclude that declaring delayed detonations *necessary* for successful explosions is premature. This does not mean that delayed detonation do not occur in general – they may still be needed to explain some peculiar events with high ejecta velocities [18] – but more work is needed to explore the range of explosion energies and nucleosynthesis products of pure turbulent deflagration explosions.

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