# Modeling Phenomena from Nature by Hyperbolic Partial Differential Equations

Oberwolfach Mini-Workshop April 11 - 17, 2021

organized by:

Christian Klingenberg (Würzburg)
Qin Li (Madison, Wisc. USA)
Marlies Pirner (Würzburg)

| | Su 11 | Mo 12 | Tue 13 | Wed 14 | Thurs 15 | Fr 16 | Sa 17 |
|---|---|---|---|---|---|---|---|
| 8 am | | breakfast | breakfast | breakfast | breakfast | breakfast | breakfast |
| 9 am | | 9 Welcome<br>9:05 Seok-Bae Yun<br>9:55 Marlies Pirner<br>10:45 break<br>11:15 Farah Kanbar | 9 Min Tang<br>9:50 break<br>12 Jonas Jackwirth | 9 Wasilij Barsukow<br>9:50 break<br>10:20 Wasilij Barsukow | 9 Eduard Feireisl<br>9:50 Simon Markfelder<br>10:40 break<br>11:10 Eva Horlebein | 9 Gi-Chan Bae<br>9:50 Sandra Warnecke<br>10:40 break<br>11:10 Eduard Feireisl | |
| 12:30 | | lunch | lunch | lunch | lunch | lunch | |
| 2:30 pm | | cake & coffee | cake & coffee | 2 pm: Oberwolfach hike | cake & coffee | cake & coffee<br>3 Qin Li | |
| 4 pm | | 4 Claudius Birke<br>4:50 break | 4 Lena Baumann<br>4:50 break<br>5:20 Philipp Edelmann | | 4 Ru-Yu Lai<br>4:50 break<br>5:20 Ru-Yu Lai | 4 break<br>4:30 Kathrin Hellmuth | |
| 6:30 pm | dinner | dinner | dinner | dinner | dinner | dinner | |

kinetic — numerics for cons. laws — theory for Euler — inverse problems

## MONDAY, APRIL 12

9:00 am **Yun, Seok-Bae** (Suwon, Korea)

Title: Entropy-Entropy production estimate for ES-BGK model with the correct Prandtl number.

Abstract: The ellipsoidal BGK model (ES-BGK model) is a model equation of the Boltzmann equation introduced to obtain the correct Prantl number in the Navier-Stokes limit. In this talk, we present our recent results on the entropy-entropy production estimate for the ES-BGK model that holds in the whole range of the Prandtl parameters -1/2< \nu< 1. Our result improves the previous entropy production estimate in that (1) the full range of Prandtl parameter including the critical case \nu=-1/2 is covered, and (2) a sharper entropy production coefficient bound is obtained.

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9:50 am **Pirner, Marlies** (Würzburg):

Title: Thermalization of a rarefied gas with total energy conservation: existence, hypocoercivity, macroscopic limit

Abstract: In this talk I will consider a kinetic relaxation model which describes the thermalization of a gas towards a Maxwellian with a background temperature. The temperature of the background is described by a heat equation with an inhomogeneity such that we have conservation of total energy. For this coupled nonlinear system, I will present existence of solutions, convergence to global equilibrium and the rigorous macroscopic limit towards a non-linear cross diffusion problem.

This is joint work with Christian Schmeiser und Gianluca Favre (both at University of Vienna).

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11:20 am **Kanbar, Farah** (Würzburg):

Title: A criterion for asymptotic preserving schemes of kinetic equations to be uniformly stationary preserving.

Abstract: In this work we are interested in the stationary preserving property of asymptotic preserving (AP) schemes for kinetic models. We introduce a criterion for AP schemes for kinetic equations to be uniformly stationary preserving (SP). Our key observation is that as long as the Maxwellian of the distribution function can be updated explicitly, such AP schemes are also SP.

This is joint work with Christian Klingenberg & Min Tang (Shanghai, China).

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4:00 pm **Birke, Claudius** (Würzburg):

Title: A low Mach two-speed relaxation scheme for the compressible Euler equations with gravity

Abstract: We present a numerical approximation of the solutions of the Euler equations with a gravitational source term. This scheme can both preserve stationary solutions and also preserves the low Mach limit to the corresponding incompressible equations. In addition we prove that our scheme guarantees ***not*** to give rise to numerical checkerboard modes in the incompressible limit.

This is joint work with Christophe Chalons (Versailles, France) & Christian Klingenberg.

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## TUESDAY, APRIL 13

9:00 am **Tang, Min** (Shanghai, China)

Titel: Individual based models for bacterial movement exhibiting Lévy-flight type movement induced by intra-cellular noise

Abstract: Kinetic-transport models that relate the intra-cellular reaction with the run-and-tumble process are considered as the correct description of microscopic level bacterial movement. Various macroscopic equations of Keller-Segel type or hyperbolic type have been derived from the pathway based kinetic-transport equations for E.coli chemotaxis. Most biological processes have a lot of noise, intrinsic or extrinsic. For E.coli chemotaxis, noise can occurs in the signaling pathways and affects the tumbling rate. We work on an individual-based model and a kinetic equation whose pathways and tumbling kernels are biologically relevant. Super diffusion can arise in the macroscopic limits, under proper scaling and conditions on the tumbling frequency as well as the form of noise. Biologically relevant pathways and tumbling kernels are considered that allows for numerical or possible experimental verifications.

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9:50 am **break**

12 noon **Jackwirth, Jonas** (Würzburg):

Title: A spectral method for solving the Korteweg-de-Vries equations

Abstract: This is part of my Master thesis under the supervision of Christian Klingenberg.

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4:00 pm **Baumann, Lena** (Würzburg):

Titel: Damping phenomena for the Vlasov-Poisson-BGK equation with small collision frequencies with a focus on Landau damping

Abstract: When Landau damping was discovered for the Vlasov-Poisson equation in the 1940s in a strictly mathematical way by the Soviet physicist Lev Landau this was a quite astonishing result for the mathematical and physicist community. Following the approach used by Landau we show that there is also a damping effect for the Vlasov-Poisson-BGK equation for small collision frequencies. In a first step we follow a paper by Wood and Ninham and show by analytical means and approximations that the damping effect can be split up into a collisional damping due to the BGK relaxation and a Landau damping part. In a second step we confirm this result by numerical examples.

I acknowledge helpful discussions with Christian Klingenberg, Marlies Pirner & Sandra Warnecke.

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4:50 am **break**

5:20 am **Edelmann, Philipp** (Los Alamos Lab, USA)

Title: Well-balanced treatment of gravity in astrophysical fluid dynamics simulations at low Mach numbers

Abstract: *Context*. Accurate simulations of flows in stellar interiors are crucial to improving our understanding of stellar structure and evolution. Because the typically slow flows are but tiny perturbations on top of a close balance between gravity and pressure gradient, such simulations place heavy demands on numerical hydrodynamics schemes.

*Aims*. We demonstrate how discretization errors on grids of reasonable size can lead to spurious flows orders of magnitude faster than the physical flow. Well-balanced numerical schemes can deal with this problem.

*Methods*. Three such schemes are applied in the implicit, finite-volume code SLH in combination with a low-Mach-number numerical flux function. We compare how the schemes perform in four numerical experiments addressing some of the challenges imposed by typical problems in stellar hydrodynamics.

*Results*. We find that the $\alpha$-$\beta$ and Deviation well-balancing methods can accurately maintain hydrostatic solutions provided that grav- itational potential energy is included in the total energy balance. They accurately conserve minuscule entropy fluctuations advected in an isentropic stratification, which enables the methods to reproduce the expected scaling of convective flow speed with the heating rate. The Deviation method also substantially increases accuracy of maintaining stationary orbital motions in a Keplerian disk on long time scales. The Cargo–LeRoux method fares substantially worse in our tests, although its simplicity may still offer some merits in certain situations.

*Conclusions*. Overall, we find the well-balanced treatment of gravity in combination with low Mach number flux functions essential to reproducing correct physical solutions to challenging stellar slow-flow problems on affordable collocated grids.

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## WEDNESDAY, APRIL 14

9:00 am **Barsukow, Wasilij** (Garching, Germany)

Title: Active flux method for nonlinear conservation/balance laws

Abstract: This talk focuses on the numerical solution of hyperbolic conservations laws (possibly endowed with a source term) using the Active Flux method. This method is an extension of the finite volume method. Instead of solving a Riemann Problem, the Active Flux method uses actively evolved point values along the cell boundary in order to compute the numerical flux. Early applications of the method were linear equations with an available exact solution operator, and Active Flux was shown to be structure preserving in such cases. For nonlinear PDEs or balance laws, exact evolution operators generally are unavailable. I will discuss strategies how sufficiently accurate approximate evolution operators can be designed which allow to make Active Flux structure preserving / well-balanced for nonlinear problems.

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10:20 am **Barsukow, Wasilij** (Garching, Germany)

Title: An informal discussion on structure preserving methods

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## THURSDAY, APRIL 15

9 am **Feireisl, Eduard**

Title: Statistical solutions to the barotropic Navier-Stokes system

Abstract: We introduce a new concept of statistical solution in the framework of weak solutions to the barotropic Navier--Stokes system with inhomogeneous boundary conditions. Statistical solution is a family of Markov operators on the set of probability measures on the data space containing the initial and the boundary data. The operators enjoy the following properties:

1. Semiflow (semigroup) property
2. The solution is deterministic for deterministic data
3. Continuity in a suitable Bregman-Wasserstein metric

This is joint work with F. Fanelli (Lyon I).

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9:50 am **Markfelder, Simon** (Cambridge, England)

Title: Convex integration applied to the compressible Euler equations

Abstract: In the past 10 years a method called convex integration has been used to prove some results concerning non-uniqueness of solutions to the compressible Euler system in multiple space dimensions. However the method used so far seems to be quite restrictive, especially in view of the initial data for which it can be applied. In this talk we sketch how convex integration works in the context of compressible Euler. Furthermore we give an idea of a possible generalization to overcome the above-mentioned restriction.

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11:20 am **Horlebein, Eva** (Würzburg):

Title: On the convergence of the invariant domain preserving numerical scheme by J.-L. Guermond and B. Popov for the complete Euler system

Abstract: In this talk I will present work in progress of proving (under suitable assumptions) the convergence of the invariant domain preserving scheme of J.-L. Guermond and B. Popov to a dissipative measure-valued solution for the full Euler equations. In a first step I introduce the aforementioned scheme and define the meaning of invariant domain preserving. Next I will talk about different kinds of convergence and explain why we chose to work with measure-valued convergence. The last part of the talk is about explaining the proposed proof. In this last part we need to first establish suitable a priori bounds, which ensure the weak-* convergence of the approximating sequences to some limit. The second step will be to prove consistency of the method. This ensures that the found limits of the approximating sequences are indeed solutions of the Euler equations. Note, that this last part is work in progress.

We acknowledge helpful discussions Eduard Feireisl (Prague), Christian Klingenberg and Simon Markfelder (Cambridge).

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4 pm **Lai, Ru-Yu** (Minneapolis, USA):

Title: Introduction to Inverse Problems and Transport Equations I

Abstract: Inverse problems are the opposites of direct problems. While one aims to find the unique effect (solution) of a given cause in the direct problems, the inverse problems can be interpreted as finding the cause of a given effect or specifying the model from certain given information of effect. In other words, the goals of inverse problems involve the reconstruction of the internal characteristics of an inaccessible region from the boundary measurements as well as the determination of coefficients appearing in the underlying equations from the input and output measurements.

In these two lectures, I will introduce some applications of inverse problems and their related mathematical backgrounds, and put a special focus on inverse problems for transport equations.

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4:50 pm **break**

5:20 pm **Lai, Ru-Yu** (Minneapolis, USA):

Title: Introduction to Inverse Problems and Transport Equations II

Abstract: see part I above

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## FRIDAY, APRIL 16

9 am **Bae, Gi-Chan** (Seoul, Korea)

Title: On the kinetic Shakhov model near a global Maxwellian

Abstract: The Shakhov model is a relaxation approximation of the Boltzmann equation proposed to overcome the deficiency of the original BGK model, namely, the incorrect production of the Prandtl number.

In this talk, we address the existence and asymptotic stability of the Shakhov model when the initial data is a small perturbation of global equilibrium. We derive a dichotomy in the coercive estimate of the linearized relaxation operator between zero and non-zero Prandtl number and observe that the linearized relaxation operator becomes more degenerate in the former case. To fill out such degeneracy and recover the full coercivity, we consider a micro-macro equation that involves non-conservative quantities.

This is joint work with Prof. Seok-Bae Yun. (Suwon, Korea).

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9:50 am **Warnecke, Sandra** (Würzburg)

Title: Numerical schemes for multi-species (quantum) BGK equations

Abstract: We consider a kinetic model of a gas mixture. Each gas species is described by a BGK model, where for N species we have N BGK terms for each species, describing the pairwise interaction of each species with every other. The special feature of our model is that we do not only consider a pure classic gas mixture nor a pure quantum gas mixture, but we also allow for a mixture of quantum and classic particles. One can show that mass, total momentum and total energy are conserved and an H-Theorem holds. We design a numerical method for this class of models and show numerical results.s

This is joint work with Christian Klingenberg & Marlies Pirner (Würzburg).

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11:20 am **Feireisl, Eduard** (Prague)

Title: Ergodic theory for energetically open compressible fluid flows

Abstract: The ergodic hypothesis is examined for energetically open fluid systems represented by the barotropic Navier-Stokes equations with general inflow/outflow boundary conditions. We show that any globally bounded trajectory generates a stationary statistical solution, which is interpreted as a stochastic process with continuous trajectories supported by the family of weak solutions of the problem. The abstract Birkhoff--Khinchin theorem is applied to obtain convergence (in expectation and a.s.) of ergodic averages for any bounded Borel measurable function of state variables associated to any stationary solution.

Finally, we show that validity of the ergodic hypothesis is determined by the behavior of entire solutions In particular, the ergodic averages converge for any trajectory provided its omega-limit set in the trajectory space supports a unique (in law) stationary solution.

This is joint work with F. Fanelli (Lyon I) and M. Hofmanova (TU Bielefeld).

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3 pm **Li, Qin** (Madison, Wisc., USA):

Title: Low rank structure in the forward and inverse kinetic theory

Abstract: Kinetic theory is a body of theory from statistical mechanics. It is useful in describing the dynamics of a large number of particles, but its high dimensional structure makes the computation infeasible. In multi-scale regimes, however, kinetic equations can be compressed: The Boltzmann equation is asymptotically equivalent to the Euler equations, and the radiative transfer equation is asymptotically equivalent to the diffusion equation. In linear algebra, this phenomenon is equivalent to a system being of low rank.

I will discuss how the low rank structure forms, and how it affects the computation. In the forward regime, inserting the low-rank structure greatly advances the computation, but in the inverse regime, the system being of low rank typically makes the problems significantly harder.

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4:30 pm **Hellmuth, Kathrin** (Würzburg):

Title: An inverse problem for chemotaxis

Abstract: In mathematical modeling of physical phenomena, one often faces the problem of determining model parameters that are not observable. Inverse problems address this problem by investigating the reconstruction of model parameters when the model input and the corresponding result of the model are known. For PDEs this means that one e.g. tries to determine a coefficient in the PDE by observing its solutions corresponding to several known initial data.

In this framework, we investigate a problem from mathematical biology: The chemotaxis equation describes the movement of bacteria in an environment with a chemical attractant on a kinetic level. Since this movement is driven by a 'run-and-tumble' process, it is of great interest to determine the tumbling kernel describing this process. It shall thus be reconstructed by the solution of the chemotaxis equation for given initial data.

In a first step, the Bayesian solution of this problem is compared to solution of the corresponding inverse problem for the Keller Segel equation which models the phenomenon on the macroscopic level.

This is joint work with Christian Klingenberg, Qin Li (Madison, Wisc.), Min Tang (Shanghai).

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