Potentialtheorie, Fall 2017/18

Exercise sheet 1

Exercise 1:

(a) Suppose that $u(\xi,\eta) = 2\xi\eta$. Find a function $v(\xi\eta)$ such that $f(z) = u(\xi,\eta) + \mathrm{i}v(\xi,\eta)$ is holomorphic.

(b) For $z = \xi + \mathrm{i}\eta$ with $|z| < 1$ we consider the Möbius transform

$$f(\zeta) = \frac{\zeta + z}{1 + \overline{z}\zeta}\,.$$

Show that f maps the unit disk onto the unit disk, and check that the inverse of f is given by

$$f^{-1}(w) = \frac{w - z}{1 - \overline{z}w}\,.$$

Aufgabe 2:

(a) Suppose that $u \in C^1(\mathbb{R}^2)$ and $\mathbf{v} \in C^1(\mathbb{R}^2)^2$. Show that the following chain rule holds:

$$\mathrm{div}(u\mathbf{v}) = \nabla u \cdot \mathbf{v} + u\mathrm{div}\mathbf{v}\,.$$

(b) Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary, $u \in C^1(\overline{\Omega})$ and $\mathbf{v} \in C^1(\overline{\Omega})^2$. Show, using the divergence theorem, that

$$\int_\Omega u\,\mathrm{div}\mathbf{v}\,\mathrm{d}x = -\int_\Omega \nabla u \cdot \mathbf{v} + \int_{\partial\Omega} u\,\nu \cdot \mathbf{v}\,\mathrm{d}s\,.$$

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