Global solutions of the Cauchy Problem to Euler-Poisson equations of two-carrier types

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Abstract. In this paper, we introduce the flux approximation coupled with the classical viscosity method to study the global entropy solutions to the Cauchy problem of the inhomogeneous Euler-Poisson equations of two-carrier types in one dimension with arbitrarily large initial data, and arbitrary adiabatic exponent $\gamma > 1$.

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1. Introduction

In this paper, we consider the following Euler-Poisson equations of two-carrier types in one dimension

$$\begin{cases} \rho_{it} + (\rho_i u_i)_x = -R(\rho_1, \rho_2), \\ (\rho_i u_i)_t + (\rho_i (u_i)^2 + P_i(\rho_i))_x = \rho_i E - \frac{\rho_i u_i}{\tau_i}, \quad i = 1, 2, \\ E_x = \rho_1 + \rho_2 - b(x), \end{cases} \tag{1.1}$$

in the region $(-\infty, +\infty) \times [0, T]$, with bounded initial data

$$(\rho_i, u_i)|_{t=0} = (\rho_{i0}(x), u_{i0}(x)), \quad \lim_{|x|\to\infty} (\rho_{i0}(x), u_{i0}(x)) = (0, 0), \quad \rho_{i0}(x) \geq 0 \tag{1.2}$$

and a condition at $-\infty$ for the electric potential

$$\lim_{x\to-\infty} E(x,t) = E_0, \quad \text{for a.e.} \quad t \in (0, \infty), \tag{1.3}$$

where T, E_0 are fixed constants, (ρ_1, u_1) and (ρ_2, u_2) are the (density, velocity) pairs for electrons ($i = 1$) and holes ($i = 2$) respectively, E is the electric potential and the given function $b(x)$ represents the impurity doping profile (cf. [**1**]-[**10**] and the references cited therein). The pressure-density relations are $P_i(\rho_i) = \frac{1}{\gamma_i}(\rho_i)^{\gamma_i}$, where $\gamma_i > 1$ correspond to the adiabatic exponents, $\tau_i > 0$ are the momentum relaxation times.

The recombination-generation rate R in (1.1) takes the form

$$R(\rho_1, \rho_2) = Q(\rho_1, \rho_2)(\rho_1 \rho_2 - 1) \tag{1.4}$$

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with a nonnegative, locally Lipschitz continuous function $Q(\rho_1,\rho_2)$ satisfying

$$0 \le Q(\rho_1,\rho_2) = \frac{q_0}{(1+\rho_1+\rho_2)}, \quad \frac{|R(\rho_1,\rho_2)|}{\rho_i} \le q_0, \quad \text{for all} \quad \rho_1>0, \rho_2>0, \tag{1.5}$$

where q_0 is a positive constant.

The Initial-boundary value problem of (1.1) with the condition (1.5) on $R(\rho_1,\rho_2)$ was first studied in [**1, 2**], where the adiabatic exponents γ_i are limited in the region $(1,3]$ to ensure the uniform L^∞ estimates of the viscosity approximation solutions (cf. the proof of **(3.9)** in[**1**]), or of the approximation solutions constructed by the Godunov scheme with fractional step (cf. [**2**]), respectively. The Cauchy problem of (1.1) for the homogeneous case ($R(\rho_1,\rho_2)=0$) was studied in [**3, 4**] for any $\gamma_i>1$, where the approximation solutions were constructed by the Lax-Friedrichs scheme and the Godunov scheme. Due to the lack of a technique to obtain the a-priori L^∞ estimate, it is a long-standing open problem to study the Cauchy problem of (1.1) by using the vanishing viscosity method. In [**11**], a L^p solution, $1<p<\infty$, was studied by using the vanishing viscosity method.

In this paper, we apply the flux approximation coupled with the classical viscosity method, introduced in [**12**], to study the global entropy solutions of the Cauchy Problem (1.1)-(1.3) and (1.5) with arbitrarily large initial data, and arbitrary adiabatic exponents $\gamma_i>1$.

We consider

$$\begin{cases} \rho_{it} + ((\rho_i - 2\delta)u_i)_x = \varepsilon\rho_{ixx} - \frac{\rho_i - 2\delta}{\rho_i}R(\rho_1,\rho_2), \\ (\rho_i u_i)_t + (\rho_i(u_i)^2 - \delta(u_i)^2 + S_i(\rho_i,\delta))_x = \varepsilon(\rho_i u_i)_{xx} + \rho_i E - \frac{\rho_i u_i}{\tau_i}, \\ E_x = (\rho_1 - 2\delta) + (\rho_2 - 2\delta) - b(x) \end{cases} \tag{1.6}$$

with the initial data

$$(\rho_i^{\varepsilon,\delta}(x,0), u_i^{\varepsilon,\delta}(x,0)) = (\rho_{i0}(x) + 2\delta, u_{i0}(x)) * G^\varepsilon, \tag{1.7}$$

where $(\rho_{i0}(x), u_{i0}(x))$ are given in (1.2), $\delta>0$ denotes a regular perturbation constant, the perturbation pressures

$$S_i(\rho_i,\delta) = \int_{2\delta}^{\rho_i} \frac{t-2\delta}{t} P_i'(t)dt, \tag{1.8}$$

G^ε is a mollifier such that $(\rho_i^{\varepsilon,\delta}(x,0), u_i^{\varepsilon,\delta}(x,0))$ are smooth and

$$\lim_{|x|\to\infty} (\rho_i^{\varepsilon,\delta}(x,0), u_i^{\varepsilon,\delta}(x,0)) = (2\delta, 0), \quad \lim_{|x|\to\infty} (\rho_{ix}^{\varepsilon,\delta}(x,0), u_{ix}^{\varepsilon,\delta}(x,0)) = (0,0). \tag{1.9}$$

The main result of this paper is given in the following:

THEOREM 1.1. *Let $P_i(\rho_i) = \frac{1}{\gamma_i}(\rho_i)^{\gamma_i}$, the initial data $0 \le \rho_{i0}(x) \in L^\infty(\mathbb{R}) \cap L^1(\mathbb{R})$, $u_{i0}(x) \in L^\infty(\mathbb{R})$, the doping profile $b(x) \in L^1(\mathbb{R})$ and τ_i be sufficiently small such that*

$$0 < \tau_i < \frac{2}{(\gamma_i+1)q_0}. \tag{1.10}$$

Then, **(I)** *for fixed $\varepsilon, \delta > 0$ and $\gamma_i > 1$, the problem (1.5)-(1.7) has a global smooth solution $(\rho_i^{\varepsilon,\delta}, u_i^{\varepsilon,\delta}, E^{\varepsilon,\delta})$ satisfying*

$$2\delta \le \rho_i^{\varepsilon,\delta} \le M(t), \quad |u_i^{\varepsilon,\delta}| \le M(t), \quad |E^{\varepsilon,\delta}| \le M(t), \quad |\rho_i^{\varepsilon,\delta}(\cdot,t) - 2\delta|_{L^1(\mathbb{R})} \le M(t), \tag{1.11}$$

where $M(t)$ is a bounded function of t, which is independent of $\varepsilon, \delta, \tau_i$, but could tend to infinity when t goes to infinity; and

(II) *there exists a subsequence of $(\rho_i^{\varepsilon,\delta}(x,t), u_i^{\varepsilon,\delta}(x,t), E^{\varepsilon,\delta}(x,t))$, which converges pointwisely to a weak entropy solution $(\rho_i(x,t), u_i(x,t), E(x,t))$ of the problem (1.1)-(1.3) as δ, ε tend to zero.*

Definition 1. *$(\rho_i(x,t), u_i(x,t), E(x,t))$ is called a weak entropy solution of the problem (1.1)-(1.3) if*

(1.12)
$$\begin{cases} \int_0^\infty \int_{-\infty}^\infty \rho_i\phi_t + (\rho_i u_i)\phi_x - R(\rho_1,\rho_2)\phi(x,t)dxdt + \int_{-\infty}^\infty \rho_{i0}(x)\phi(x,0)dx = 0, \\ \int_0^\infty \int_{-\infty}^\infty \rho_i u_i\phi_t + (\rho_i u_i^2 + P_i(\rho_i))\phi_x + (\rho_i E(x,t) - \frac{\rho_i u_i}{\tau})\phi(x,t)dxdt \\ + \int_{-\infty}^\infty \rho_{i0}(x)u_{i0}(x)\phi(x,0)dx = 0, \\ \int_0^\infty \int_{-\infty}^\infty E(x,t)\phi_x + (\rho_1+\rho_2-b(x))\phi(x,t)dxdt \end{cases}$$

holds for $i = 1, 2$, all test function $\phi \in C_0^1(R \times R^+)$ and

$$\begin{aligned} &\int_0^\infty \int_{-\infty}^\infty \eta(\rho_i,m_i)\phi_t + q(\rho_i,m_i)\phi_x \\ &-(R(\rho_1,\rho_2)\eta(\rho_i,m_i)_{\rho_i} - (\rho_i E(x,t) - \tfrac{\rho_i u_i}{\tau_i})\eta(\rho_i,m_i)_{m_i})\phi(x,t)dxdt \geq 0 \end{aligned} \tag{1.13}$$

holds for any non-negative test function $\phi \in C_0^\infty(R\times R^+ - \{t=0\})$, where $m_i = \rho_i u_i$ and (η, q) is a pair of convex entropy-entropy flux of system (1.1).

2. Proof of Theorem 1.1

In this section, we shall prove Theorem 1.1. Multiplying the first two equations of (1.6) by $(\frac{\partial w_i}{\partial \rho_i}, \frac{\partial w_i}{\partial m_i})$ and $(\frac{\partial z_i}{\partial \rho_i}, \frac{\partial z_i}{\partial m_i})$, respectively, we have

$$\begin{aligned} &w_{it} + \lambda_{i2}^\delta w_{ix} = \varepsilon w_{ixx} + \tfrac{2\varepsilon}{\rho_i}\rho_{ix}w_{ix} - \tfrac{(\gamma_i+1)\varepsilon}{2}(\rho_i)^{\frac{\gamma_i-5}{2}}\rho_{ix}^2 \\ &-((\rho_i)^{\theta_i} - \tfrac{m_i}{\rho_i})\tfrac{\rho_i-2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i} - \tfrac{u_i}{\tau_i} + E \\ &= \varepsilon w_{ixx} + \tfrac{2\varepsilon}{\rho_i}\rho_{ix}w_{ix} - \tfrac{(\gamma_i+1)\varepsilon}{2}(\rho_i)^{\frac{\gamma_i-5}{2}}\rho_{ix}^2 \\ &-(\tfrac{\theta_i-1}{2}w_i + \tfrac{\theta_i+1}{2}z_i)\tfrac{\rho_i-2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i} - \tfrac{w_i-z_i}{2\tau_i} + E \end{aligned} \tag{2.1}$$

and

$$\begin{aligned} &z_{it} + \lambda_{i1}^\delta z_{ix} = \varepsilon z_{ixx} + \tfrac{2\varepsilon}{\rho_i}\rho_{ix}z_{ix} - \tfrac{(\gamma_i+1)\varepsilon}{2}(\rho_i)^{\frac{\gamma_i-5}{2}}\rho_{ix}^2 \\ &-((\rho_i)^{\theta_i} + \tfrac{m_i}{\rho_i})\tfrac{\rho_i-2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i} + \tfrac{u_i}{\tau_i} - E \\ &= \varepsilon z_{ixx} + \tfrac{2\varepsilon}{\rho_i}\rho_{ix}z_{ix} - \tfrac{(\gamma_i+1)\varepsilon}{2}(\rho_i)^{\frac{\gamma_i-5}{2}}\rho_{ix}^2 \\ &-(\tfrac{\theta_i+1}{2}w_i + \tfrac{\theta_i-1}{2}z_i)\tfrac{\rho_i-2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i} + \tfrac{w_i-z_i}{2\tau_i} - E, \end{aligned} \tag{2.2}$$

where

$$\lambda_{i1}^\delta = \frac{m_i}{\rho_i} - \frac{\rho_i-2\delta}{\rho_i}\sqrt{P_i'(\rho_i)}, \quad \lambda_{i2}^\delta = \frac{m_i}{\rho_i} + \frac{\rho_i-2\delta}{\rho_i}\sqrt{P_i'(\rho_i)} \tag{2.3}$$

are two eigenvalues of the approximation system (1.6), $m_i = \rho_i u_i$ denote the momentums and (w_i, z_i) are the corresponding Riemann invariants given by

$$z_i(\rho_i, u_i) = \frac{1}{\theta_i}(\rho_i)^{\theta_i} - \frac{m_i}{\rho_i}, \quad w_i(\rho_i, u_i) = \frac{1}{\theta_i}(\rho_i)^{\theta_i} + \frac{m_i}{\rho_i}, \quad \theta_i = \frac{\gamma_i - 1}{2}. \tag{2.4}$$

Applying the maximum principle to the first equation in (1.6), we first have the a-priori estimates $\rho_i \geq 2\delta$.

Integrating the both sides of the first equation in (1.6) over $R \times [0, t]$, we obtain

$$\begin{aligned} &\textstyle\int_{-\infty}^{\infty} \rho_i(x,t) - 2\delta dx = \int_{-\infty}^{\infty} \rho_{i0}^{\varepsilon,\delta}(x,0) - 2\delta dx - \int_0^t \int_{-\infty}^{\infty} \frac{\rho_i - 2\delta}{\rho_i} R(\rho_1, \rho_2) dx dt \\ &\textstyle\leq \int_{-\infty}^{\infty} \rho_{i0}(x,0)dx + \int_0^t \int_{-\infty}^{\infty} (\rho_i - 2\delta)\frac{Q(\rho_1,\rho_2)}{\rho_i} dx dt \\ &\textstyle\leq M_1 + q_0 \int_0^t \int_{-\infty}^{\infty} (\rho_i - 2\delta) dx dt \end{aligned} \tag{2.5}$$

and thus

$$\int_{-\infty}^{\infty} \rho_i(x,t) - 2\delta dx \leq M_2 e^{q_0 t}, \tag{2.6}$$

where M_i are suitable positive constants, which depend only the initial data, but are independent of ε and δ.

By integrating the third equation in (1.6), we obtain

$$|E| = |E_0 + \int_{-\infty}^{x} (\rho_1(x,t) - 2\delta) + (\rho_2(x,t) - 2\delta) - b(x) dx| \leq M_3 e^{q_0 t}. \tag{2.7}$$

We make the transformation

$$w_i = W_i + \beta e^{\alpha t}, \quad z_i = Z_i + \beta e^{\alpha t}, \tag{2.8}$$

where

$$\beta = max\{1, |w_{io}(x)|_{L^\infty}, |z_{io}(x)|_{L^\infty}\}, \quad \alpha = max\{q_0, \theta_i q_0 + M_3\}. \tag{2.9}$$

Since

$$(\rho_i)^{\theta_i} = \frac{\theta_i}{2}(w_i + z_i) = \frac{\theta_i}{2}(W_i + Z_i + 2\beta e^{\alpha t}),$$

$$u_i = \frac{m_i}{\rho_i} = \frac{1}{2}(w_i - z_i) = \frac{1}{2}(W_i - Z_i)$$

and

$$-\alpha\beta e^{\alpha t} + q_0 \theta_i \beta e^{\alpha t} \pm E \leq 0,$$

then (2.1), (2.2) can be rewritten as follows:

(2.10)

$$\begin{aligned} &W_{it} + \lambda_{i2}^{\delta} W_{ix} = \varepsilon W_{ixx} + \tfrac{2\varepsilon}{\rho_i}\rho_{ix}W_{ix} - \alpha\beta e^{\alpha t} - \tfrac{(\gamma_i+1)\varepsilon}{2}(\rho_i)^{\frac{\gamma_i - 5}{2}}\rho_{ix}^2 \\ &-(\tfrac{\theta_i - 1}{2}W_i + \tfrac{\theta_i+1}{2}Z_i + \theta_i\beta e^{\alpha t})\tfrac{\rho_i - 2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i} - \tfrac{W_i - Z_i}{2\tau_i} + E \\ &\leq \varepsilon W_{ixx} + \tfrac{2\varepsilon}{\rho_i}\rho_{ix}W_{ix} - (\tfrac{\theta_i - 1}{2}\tfrac{\rho_i - 2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i} + \tfrac{1}{2\tau_i})W_i + (\tfrac{1}{2\tau_i} - \tfrac{\theta_i+1}{2}\tfrac{\rho_i - 2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i})Z_i \end{aligned}$$

and

(2.11)
$$
\begin{aligned}
&Z_{it} + \lambda_{i1}^{\delta} Z_{ix} = \varepsilon Z_{ixx} + \tfrac{2\varepsilon}{\rho_i}\rho_{ix} Z_{ix} - \alpha\beta e^{\alpha t} - \tfrac{(\gamma_i+1)\varepsilon}{2}(\rho_i)^{\frac{\gamma_i-5}{2}}\rho_{ix}^2 \\
&-(\tfrac{\theta_i+1}{2}W_i + \tfrac{\theta_i-1}{2}Z_i + \theta_i\beta e^{\alpha t})\tfrac{\rho_i-2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i} + \tfrac{W_i-Z_i}{2\tau_i} - E \\
&\le \varepsilon Z_{ixx} + \tfrac{2\varepsilon}{\rho_i}\rho_{ix} Z_{ix} - (\tfrac{\theta_i-1}{2}\tfrac{\rho_i-2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i} + \tfrac{1}{2\tau_i})Z_i + (\tfrac{1}{2\tau_i} - \tfrac{\theta_i+1}{2}\tfrac{\rho_i-2\delta}{\rho_i}\tfrac{R(\rho_1,\rho_2)}{\rho_i})W_i,
\end{aligned}
$$

where the coefficient functions before Z_i in (2.10) and W_i in (2.11)

$$
\frac{1}{2\tau_i} - \frac{\theta_i+1}{2}\frac{\rho_i-2\delta}{\rho_i}\frac{R(\rho_1,\rho_2)}{\rho_i} \ge \frac{1}{2\tau_i} - \frac{\theta_i+1}{2}q_0 > 0 \tag{2.12}
$$

due to the conditions (1.5) and (1.10).

From (2.8), we have $W_i(x,0) \le 0, Z_i(x,0) \le 0$, so, we may apply the maximum principle to (2.10) and (2.11) to obtain the estimates $W_i(x,t) \le 0, Z_i(x,t) \le 0$. Therefore we have the following estimates

$$
w_i(x,t) \le \beta e^{\alpha t}, \quad z_i(x,t) \le \beta e^{\alpha t}, \tag{2.13}
$$

which deduce the a-priori estimates in (1.11).

To prove the part (I) in Theorem 1.1, we may first obtain a local solution of the Cauchy problem (1.5)-(1.7) by applying the general contracting mapping principle to an integral representation of (1.6). After we have the estimates in (1.11), the standard method on nonlinear parabolic system could help us to extend the local time step by step and deduce a global solution (cf. [**13**]). So, Part (I) of Theorem 1.1 is proved.

To prove the part (II) in Theorem 1.1, since $\eta(\rho_i^{\varepsilon,\delta}, u_i^{\varepsilon,\delta})_t + q(\rho_i^{\varepsilon,\delta}, u_i^{\varepsilon,\delta})_x$ are compact in $H^{-1}_{loc}(R\times R^+)$, for any weak entropy-entropy flux pair $(\eta(\rho_i,u_i), q(\rho_i,u_i))$ of system (1.1), as proved in [**12**], then by applying the compactness frameworks given in [**14**] for $1<\gamma<3$ and in [**15**] for $\gamma \ge 3$, we can easily prove the pointwise convergence $(\rho_i^{\varepsilon,\delta}(x,t), u_i^{\varepsilon,\delta}(x,t)) \to (\rho_i(x,t), u_i(x,t))$ as ε,δ go to zero.

To deduce the pointwise convergence $E^{\varepsilon,\delta}(x,t) \to E(x,t)$, we first prove the following lemma:

Lemma 2.1. *Both $E^{\varepsilon,\delta}(x,t)_t$ and $E^{\varepsilon,\delta}(x,t)_x$ are compact in $H^{-1}_{loc}(R\times R^+)$.*

Proof of Lemma 2.1. From the bounded estimate (2.7), we have that $E^{\varepsilon,\delta}(x,t)_x$ are bounded in $W^{-1}_{loc}(R\times R^+)$. Moreover, from the L^1 estimates in (2.6), we have from the third equation in (1.6) that $|E_x^{\varepsilon,\delta}|_{L^1}$ are bounded and so $E_x^{\varepsilon,\delta}$ are compact in $H^{-1}_{loc}(R\times R^+)$ by using the Murat's lemma [**16**].

Using the third and the first equations in (1.6), we have

(2.14)
$$
\begin{aligned}
E_t^{\varepsilon,\delta} &= \textstyle\int_{-\infty}^{x} \rho_{1t}(y,t) + \rho_{2t}(y,t)dy \\
&= -\textstyle\sum_{i=1}^{2}((\rho_i-2\delta)u_i + \varepsilon\rho_{ix} - \int_{-\infty}^{x}\frac{\rho_i(y,t)-2\delta}{\rho_i(y,t)}R(\rho_1(y,t),\rho_2(y,t))dy),
\end{aligned}
$$

which being locally bounded in $L^1(R\times R^+)$. Thus $E_t^{\varepsilon,\delta}$ are compact in $H^{-1}_{loc}(R\times R^+)$ by using the Murat's lemma [**16**] again. Lemma 2.1 is proved.

We may apply the Div-Curl lemma to the pairs of functions

$$
(0, E^{\varepsilon,\delta}), \quad (E^{\varepsilon,\delta}, 0), \tag{2.15}
$$

to obtain

$$\overline{E^{\varepsilon,\delta}} \cdot \overline{E^{\varepsilon,\delta}} = \overline{(E^{\varepsilon,\delta})^2}, \tag{2.16}$$

which deduces the pointwise convergence of $E^{\varepsilon,\delta}(x,t)$. Thus the part (II), and so Theorem 1.1, is proved.

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