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Topics in Noncommutative Differential Geometry

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The leitmotiv of these lectures is noncommutative principal $U(1)$-bundles and associated line bundles. The only prerequisite is some linear algebra and a basic knowledge of C^*-algebra theory. In the first part of the minicourse I will give a brief introduction to Hopf-Galois theory and its applications, from field extensions to principal group actions. I will then recall Woronowicz' definition of compact quantum group and the notion of noncommutative principal bundle. When the structure group is $U(1)$, there is a construction due to Pimsner that allows to get the total space of a "bundle" (more precisely, a strongly graded C^*-algebra) from the base space and a noncommutative "line bundle" (a self-Morita equivalence bimodule). As an example of this construction, I will discuss the bundle quantum lens space $\longrightarrow$ quantum weighted projective space (from a joint work with G. Landi and F. Arici). Another peculiar example is the noncommutative torus, that for a general value of the deformation parameter has no non-trivial self-Morita equivalence bimodules. One can still derive Connes-Rieffel imprimitivity bimodules from some kind of principal bundle, but at the price of working with non-associative algebras. Motivated by this example, last part of the minicourse will be a peek into the realm of quasi-associative algebras. I will discuss the theory of cochain quantization and its applications, from Albuquerque-Majid example of octonions, to line bundles on the noncommutative torus (this is a joint work with G. Fiore and D. Franco).