Abstracts TV Conference

Evgueni Abakoumov

Title: Chui's conjecture and rational approximation

Abstract: C. K. Chui conjectured in 1971 that the average gravitaional field strength in the unit disk due to unit point masses on its boundary was the smallest when these point masses were equidistributed on the circle. We will present an elementary solution to some weighted versions of this problem, and discuss related questions concerning approximation of holomorphic functions by simple partial fractions. This is joint work with A. Borichev and K. Fedorovskiy.

Nicola Arcozzi

Title: The capacity of the Dobinski set

Abstract: A number in $[0,1]$ belongs to the Dobinski set if it can be 'very well' approximated by dyadic rationals. A. Dayan, J. L. Fernández, M. J. González and others have recently obtained sharp information on the Hausdorff dimension of this and related sets, and raised a question on its logarithmic capacity. The answer to the question is joint work with N. Chalmoukis.

Kari Astala

Title: Burkholder Area Formula

Abstract: Burkholder functional B_p provides a fascinating bridge between martingale inequalities, singular integrals and vector valued calculus of variations. It gives a particularly interesting candidate to test Morrey's conjecture in two dimensions, i.e. whether every rank-one convex integrand or functional in $\mathbb{R}^{2\times 2}$ is quasiconvex.

In this joint work, with D. Faraco, A. Guerra, J. Kristensen and A. Koski, we study the local properties of the functional. Recalling that $B_2(A) = -\det(A)$, we prove a "Burkholder-version" of the classical area theorem. This implies, for instance, that B_p is quasiconvex in the space of Sobolev maps with $B_p(Df) \leq 0$. The result is also a key to minimization properties of several energy functionals in non-linear elasticity, where, to avoid cavitation and interpenetration of matter, natural minimisers are Sobolev homeomorphisms.

Alexander Borichev

Title: The Szegö minimum problem

Abstract: We study the Szegö-type weighted polynomial approximation problem. We characterize the measures with exponential and power decay of the

approximation error. One of the motivation is the rigidity of stationary processes with under some restrictions on the values and on the spectral measure support.

Oliver Dragicevic

Title: The p-ellipticity condition for non-divergence-form operators

Abstract: In a joint work with A. Carbonaro we applied the notion of p-ellipticity for a study of generalized Schrodinger operators with complex coefficients, for which we proved bilinear $L^p \times L^q \to L^1$ estimates with p, q being conjugate exponents. Such inequalities admit applications to holomorphic functional calculus. We also discuss a similar result for yet more general operators, namely, those involving first-order perturbations. As shown recently by A. Poggio, this requires a suitable extension of p-ellipticity.

Konstantin Dyakonov

Title: Extreme and strongly extreme points in subspaces of H^∞.

Abstract: Given a normed vector space, the extreme points of its unit ball are of traditional interest. Among these, the so-called strongly extreme points are also worth studying. In H^∞, the strongly extreme points are known to be the inner functions, while the (usual) extreme points are the unit-norm functions for which the appropriate logarithmic integral diverges. We show that similar characterizations remain valid for subspaces of finite codimension in H^∞. Also, we find out to what extent a non-inner function can differ from a strongly extreme point.

Guy David

Title: Absolutely continuous harmonic measure on Cantor sets.

Abstract: The lecture will focus on two counterexemples where the elliptic measure on a Cantor set of the plane is proportional to the Hausdorff measure. For the first one (with S. Mayboroda), the Cantor set is the one quarter set of dimension 1, but the operator is $\operatorname{div} a\nabla$ for a scalar function a; for the second one (with C. Jeznach and A. Julia) the operator is the Laplacian, but the Cantor set is slightly asymmetric and of dimension smaller than 1.

Haakan Hedenmalm

Title: Hyperbolic Fourier series and the Klein-Gordon equation

Abstract: The notion that pretty much any function or distribution on the extended real line may be expanded in a hyperbolic Fourier series was suggested in Hedenmalm, Montes-R (Annals 2011). Here, a hyperbolic Fourier series is defined as a sum of $a_n\exp(i\pi nt)$ and $b_n\exp(-i\pi n/t)$,where n runs over the integers (we typically agree that $b_0=0$). This is actually true in an even more general setting of ultradistributions, and the coefficients are uniquely determined by the (ultra)distribution in question. This gives then rise to a biorthogonal system, which consists of smooth functions with decay $O(t^{-2})$ at infinity.The biorthogonal system may be evaluated at the origin in terms of a lattice counting function on spheres in 4D Euclidean space, and a conjecture suggests that the biorthogonal system should behave essentially like complex exponentials near the origin. The proof of the existence of the hyperbolic Fourier series expansion is based on solving a Dirichlet problem on a certain surface, and successive application of Schwarz reflection along circles. The hyperbolic Fourier series expansion is intimately connected with the Fourier interpolation formulae of Radchenko and Viazovska (Publ Math IHES 2019), and the thing connecting the two are theta power twisting of the hyperbolic Fourier series and a certain transform of Fourier type which may be named after Erwin Schrodinger as it arises from solving the Schrodinger equation. The hyperbolic Fourier series is connected also with the Klein-Gordon equation in the (1+1)D setting. The biorthogonal system allows us to supply solutions to the discretized Goursat problem along the boundary of characteristics in a spacelike cone, which equal 1 at one point and vanish elsewhere on the discretized boundary. This system is analogous to the appropriate translates of the cardinal sine function in the context of the Paley-Wiener space (which equals 1 at 0 and vanishes at nonzero multiples of π). Remark: This reports on a joint project with A. Bakan, A. Montes-R, D. Radchenko, M. Viazovska.

Irina Holmes-Fay

Title: Two-weight bounds for paraproducts and sparse operators

Abstract: We investigate Bloom-type bounds for paraproducts via a new type of weighted sparse operator, and discuss connections with the square function.

Nikolai Nikolski

Title: How much does it cost to transport the frame signs on a homogeneous space?

Abstract: The sign distributions for Bessel sequences, frames and Riesz bases (u_k) are studied in L^2 spaces over the spaces of homogeneous type $\Omega = (\Omega, \rho, \mu)$. Under some relations between three basic metric-measure parameters of Ω, we obtain asymptotics for the mass moving norms $\|u_k\|_{KR}$ (Kantorovich-Rubinstein), as well as for singular numbers of the Lipschitz and Hajlasz-Sobolev embeddings. Our main observation shows that, quantitatively, the rate of the convergence $\|u_k\|_{KR} \longrightarrow 0$ depends on Bernstein-Kolmogorov n-widths of a compact set of Lipschitz functions, and the latter ones - on an interplay between geometric doubling and measure doubling/halving exponents. The 'more homogeneous' is the space, the sharper are the results.

Adam Osekovski

Title: Inequalities for differentially subordinate martingales: some recent progress

Abstract: Differentially subordinate martingales arise naturally in the context of stochastic integration, and estimates for these processes play a prominent role in the study of tight inequalities for wide classes of Fourier multipliers. As evidenced in numerous papers, these stochastic bounds can be investigated efficiently with the use of the Bellman function method. The purpose of the talk is to survey some recent progress in this direction.

Alexei Poltoratski

Title:

Abstract:

Sandra Pott

Title: Matrix Weights via Bloom BMO

Abstract: The theory of matrix Muckenhoupt weights was initiated by Nazarov, Treil, and Volberg in the late 90's and has developed rapidly since then, particularly during the last few years. At the same time, the theory of Bloom BMO spaces has been developed in various settings. In the talk, I want to show how these developments are connected, and how this connection, the 'Bloom correspondence', can be used to prove new estimates.

Alexander Pushnitsky

Title: Hankel operators with band spectra

Abstract: I will discuss spectral properties of bounded self-adjoint Hankel operators H, realised as integral operators on the positive semi-axis, that commute with dilations by a fixed factor. In analogy with the spectral theory of periodic Schroedinger operators, the Hankel operators H of this class admit the Floquet-Bloch decomposition, which represents H as a direct integral of certain compact fiber operators. As a consequence, operators H have band spectra (the spectrum of H is the union of disjoint intervals). A striking feature of this model is that flat bands (i.e. intervals degenerating into points, which are eigenvalues of infinite multiplicity) may co-exist with non-flat bands; I will discuss some simple explicit examples of this nature. The spectral analysis of this class of Hankel operator is based on the theory of elliptic functions; if time permits, I will explain this connection. This is joint work in progress with Alexander Sobolev (University College London).

Guillermo Rey

Title: Greedy approximation algorithms for sparse collections

Abstract: I'll describe a greedy algorithm that approximates the Carleson constant of a collection of general sets. The approximation has a logarithmic loss in a general setting, but is optimal up to a constant with only mild geometric assumptions. Thconstructive nature of the algorithm gives additional information about the almost-disjoint structure of sparse collections. Some of the applications of the algorithm will be explored.

Eero Saksman

Title: Random conformal weldings revisited.

Abstract: We consider direct welding constructions for SLE-type loops for small γ-parameters of the independent quantum wedges. The talk is based on joint work with Antti Kupiainen and Michael McAuley.

Kristina Skreb

Title: Bilinear embedding in Orlicz spaces for divergence-form operators with complex coefficients

Abstract: We will discuss a bi(sub)linear embedding for semigroups generated by non-smooth complex-coefficient elliptic operators in divergence form and for certain mutually dual pairs of Orlicz-space norms. This generalizes a result by Carbonaro and Dragičević from power functions to more general Young functions that still behave like powers. To achieve this, we generalize a classic Bellman function constructed by Nazarov and Treil. The talk is based on joint work with Vjekoslav Kovač.

Leonid Slavin

Title: Monotone rearrangement and Bellman functions for VMO with generalized Campanato norm

Abstract: We consider a quantitative version of VMO on an interval, equipped with a Campanato-type norm, and prove that monotone rearrangement does not increase the norm in this space. This allows us to compute Bellman functions for a family of integral functionals in this setting. Such functions are non-autonomous, in the sense that the length of the interval explicitly enters as one of the three Bellman variables. This is joint work with Pavel Zatitskii.

Joseph Slote

Title: Noncommutative Bohnenblust-Hille Inequalities with Applications to Quantum Observable Learning

Abstract: Following Eskenazis and Ivanisvili ('22), we explain how Bohnenblust–Hille (BH) inequalities for a given product space enable efficient learning of low-degree polynomials over that space from random samples. We then prove BH inequalities for finite-dimensional quantum operators (Hermitian matrices) by reducing to various commutative cases. Ultimately we recover a fast learning algorithm for qubit and qudit operators, even ones of unbounded degree in some settings. But all is not well: an important version of the result requires a classical BH inequality for products of cyclic groups, which was heretofore unexplored...

Xavier Tolsa

Title: Harmonic measure and Hausdorff measures.

Abstract: In this talk I will survey the results obtained in recent years regarding the connection between harmonic measure, Hausdorff measures, and rectifiability, which plays a prominent role in the codimension 1 case. I will also review some open questions and partial results in fractional codimensions, where the situation is not well understood. For example, a challenging question consists in finding when harmonic measure can be mutually absolutely continuous with respect to a fractional Hausdorff measure. In the talk I will mainly focus in the particular case of AD-regular sets of codimension larger than one, where there have been some recent advances in connection with an old question posed by A. Volberg.

Igor Verbitsky

Title: Harmonic measure and estimates of positive solutions for Schrödinger operators in uniform domains

Abstract: We will present global two-sided estimates for positive solutions $u>0$ of the equation

$$\begin{cases} -\Delta u = \omega u & \text{in } \Omega, \\ u = f & \text{on } \partial\Omega, \end{cases}$$

in a bounded uniform domain $\Omega \subset \mathbb{R}^n$, where $\omega \geq 0$ is a locally finite Borel measure in Ω, and $f \geq 0$. These results give sharp exponential bounds for the harmonic measure associated with the Schrödinger operator $-\Delta - \omega$ on Ω. This talk is based on joint work with Michael Frazier.

Alexander Volberg

Title: Dimension free Bohnenblust—Hille and Remez inequalities

Abstract: It turned out that to learn a large matrix in a few queries one can use the BH inequality on cyclic group of N-th roots of unity, the spectrum of blocks forming the Heisenberg—Weyl matrix basis. One way to prove BH in this setting is to have a dimension free Remez inequality. I will sketch the proof of one such dimension free discrete Remez inequality. This is a joint work with Joe Slote and Haonan Zhang.