An Outlook on Analysis at the End of AvH

Abstracts

Evgueni Abakoumov

Title: Riesz Basis Stability for Perturbed Exponentials in Paley–Wiener Spaces

Abstract: We address stability questions for exponential systems in Paley–Wiener spaces associated with subsets of the real line, focusing mainly on the case of unions of two intervals. This is an ongoing joint project with Yu. Belov and M. Mironov.

Alexander Borichev

Title: Sharp Invertibility in Quotient Algebras of H^∞

Abstract: Given an inner function Θ, we study the quotient algebra $H^\infty/\Theta H^\infty$, and more precisely, the invertibility control for its elements. We characterize the classes of Θ corresponding to different kinds of control.

Jose Manuel Conde Alonso

Title: Haar shifts for general filtrations

Abstract: Haar shifts are dyadic models of Calderón-Zygmund operators that represent them faithfully. The simplest Haar shifts, usually called Haar multipliers, can be viewed as martingale transforms, and can be studied both with analytic tools and with purely probabilistic ones. This does not apply to S, the linear operator defined on the Haar basis $\{h_I\}_{I \in \mathcal{D}}$ by

$$h_{I_+} \mapsto h_{I_-} \quad \text{and} \quad h_{I_-} \mapsto h_{I_+}.$$

S is a precise discrete analogue of the classical Hilbert transform $\mathcal{H}$, which allows one to deduce sharp weighted inequalities for $\mathcal{H}$ by just looking at it. In this talk, we will report on recent progress in interpreting all Haar shifts as generalized martingale transforms, including some results in the dyadic setting with general measures. The talked is based on joint and ongoing works with Tainara Borges, Francesco D'Emilio, Jill Pipher, and Nathan Wagner.

Oliver Dragicevic

Title: The p-ellipticity condition for systems of partial differential equations with complex coefficients

Abstract: We extend the concept of p-ellipticity for (single) elliptic operators, that we introduced in 2016, to the case of systems of elliptic equations with complex coefficients. The p-ellipticity condition from this paper turns out to be equivalent, up to a factor depending on p, to a condition for systems introduced (and equally named) by Dindoš, Li and Pipher in 2021. We prove several key properties of p-ellipticity akin to those that have been known to hold in the scalar case. In some respects, however, the vector case fundamentally differs from the scalar one. As an application we establish contractivity on L^p of the operator semigroup generated by associated elliptic operators subject to Dirichlet, Neumann or mixed boundary conditions.

Konstantin Dyakonov

Title: Carleson-type embeddings with closed range

Abstract: We characterize the Carleson measures on the disk for which the associated embedding operator has closed range. This is done in both Hardy and Bergman space settings.

Johanna Fladung

Title: The optimal bound e in a dyadic version of Uchiyama's Lemma

Abstract: We discuss a dyadic version of Uchiyama's Lemma obtained in joint work with Stefanie Petermichl. In the continuous setting this lemma plays a key role in establishing the best known upper bound for the operator norm of the Carleson Embedding from the Hardy-Hilbert space H^2 into $L^2(\mu)$ with μ being a Carleson measure. It states, that for a bounded subharmonic function $\varphi \leq 0$ on the unit disc $\mathbb{D} \subseteq \mathbb{C}$ and for the measure

$$d\mu = \Delta\varphi(z) \log \frac{1}{|z|} dA(z)$$

the space $H^2(\mathbb{D})$ embeds into $L^2(\mu)$:

$$\int_{\mathbb{D}} |f(z)|^2 d\mu(z) \leq e\|\varphi\|_\infty \|f\|_{H^2}^2 \qquad \forall f \in H^2(\mathbb{D}).$$

It was conjectured by Sergei Treil that the prefactor e is sharp. Departing from the classical case in complex analysis, we consider a dyadic analogue. Analyticity is replaced by a condition on orthonormal martingale differences of a real and an imaginary part. In this setting, a sufficiently large constant of the analogous embedding remains e. But in contrast to the continuous case, we can show that e is optimal. Moreover, we may deduce a dyadic version of the reproducing kernel thesis for the embedding theorem by utilizing the dyadic Uchiyama Lemma.

Irina Holmes

Title: The Bellman Function for Level Sets of Sparse Operators

Abstract: We investigate weak-type (1,1) boundedness of sparse operators with respect to Lebesgue measure. Specifically, we find the Bellman function maximizing level sets of sparse operators (localized to an interval) and use this to find the exact weak-(1,1) norm of these sparse operators.

Spyridon Kakaroumpas

Title: Matrix weighted Fefferman–Stein inequalities and two-matrix weighted biparameter product BMO

Abstract: The classical Fefferman–Stein inequalities on vector valued estimates for the maximal function and their weighted versions are an indispensable tool for estimating biparameter singular integral operators and their commutators. In this talk, we discuss recent work establishing a version of the Fefferman–Stein inequalities in the matrix weighted setting. We rely on the machinery of convex body valued analysis developed by M. Bownik and D. Cruz-Uribe. In particular, we prove a general vector valued version of their extrapolation method with matrix weights, which yields the Fefferman–Stein inequalities as a special case.

As a natural application, we present the development of a theory of two-matrix weighted biparameter product BMO. This includes a version of H^1-BMO duality and upper bounds for bicommutators in the matrix weighted setting. This extends earlier work of I. Holmes, S. Petermichl and B. Wick as well as J. Isralowitz, S. Pott and S. Treil in the scalar weighted, respectively one parameter setting. The talk concludes with some open questions in this line of investigations.

The talk is based on joint works with Odí Soler i Gibert (Universitat Politècnica de Catalunya).

Constanze Liaw

Title: Singular Spectrum under a Wide Class of Perturbations

Abstract: For bounded self-adjoint operators A and K on a separable Hilbert space, consider perturbed operators of the form $A + K$. We present restrictions on the singular spectrum under trace class and more general perturbations. Some of the results are for a one-parameter family of perturbations $A + tK$, as the real parameter t varies. To the best of our knowledge, these are the first statements on the singular spectrum under infinite rank perturbations. This is joint work with Eero Saksman and Sergei Treil.

Emiel Lorist

Title: Sparse domination: Now with extra cancellation

Abstract: Sparse domination has become a standard tool in harmonic analysis over the past decade. It provides a way to control a complicated operator (e.g. a Calderón–Zygmund operator) by a simple, positive expression called a sparse operator. Once such a domination principle is available, (sharp) weighted L^p bounds for $p \in (1, \infty)$ follow. In this sense, sparse domination decouples the operator-specific analysis from the subsequent norm estimates. As is well-known, many central operators in harmonic analysis fail to be bounded on L^1. Instead, one has endpoint substitutes such as boundedness from the Hardy space H^1 to L^1. Such results have, so far, been beyond the scope of sparse domination: H^1 is inherently cancellative, while sparse operators are positive and therefore fail to capture cancellative properties. In this talk, I will explain how to build a cancellative version of sparse domination, extending sparse domination techniques to norm estimates in (weighted) Hardy spaces. This is based on joint work with José Conde-Alonso and Guillermo Rey (Universidad Autónoma de Madrid).

Nicolas Nikolski

Title: Variations on a theme by Alfred Haar

Abstract: The subject of the talk takes part of the (unsolved) Completeness Dilation Problem (DCP); one of its equivalent forms requires to determine the dilation cyclic systems $(f(nx))_{n \geq 1}$ on the space $L^2_{odd}(-1, 1)$ (extended 2-periodically on).

Two elementary approaches to certain partial cases of the problem will be presented, including the Haar original claim (1925). The principal equivalent form of the DCP as the problem of cyclic vectors of the Hardy space $H^2(\mathbb{D}_2^\infty)$ on the Hilbert multidisc $\mathbb{D}_2^\infty$ is also presented. Several concrete examples will be discussed.

Javier Parcet

Title: Fourier and Schur idempotents

Abstract: What happens to an L_p function when one truncates its Fourier transform to a domain? This is in the root of foundational problems in harmonic analysis. Fefferman's celebrated theorem for the ball (1971) imposes that, to preserve L_p-integrability, the boundary of such domain must be flat. What if we truncate on a curved space like a Lie group? What do we mean by "boundary flatness" in that case? And if we truncate the entries of a given matrix? What happens with the singular numbers of it or with its Schatten

p-norm?

We will fully characterize the local geometry of such L_p-preserving truncations for these apparently unrelated problems, in terms of a surprising lax notion of boundary flatness. The matrix ones are all diffeomorphic variations of a fundamental map: the triangular projection or matrix Hilbert transform. The Lie group ones are all modeled on one of three fundamental examples: the classical Hilbert transform, and two new examples of Hilbert transforms that we call affine and projective. This vastly generalizes Fefferman's theorem to nontrigonometric and noncommutative scenarios. It confirms that Schur multipliers share profound similarities with Euclidean Fourier multipliers —even in the lack of a Fourier transform— and complete for Lie groups a longstanding search of Fourier L_p-idempotents.

<div align="right">Joint work with M. de la Salle and E. Tablate.</div>

Jill Pipher

Title: Carleson measures and regularity of solutions to elliptic/parabolic operators

Abstract: We will discuss the significant role of Carleson measures in elliptic and parabolic PDE, tracing the historical developments in this subject.

In 1958, Lennart Carleson introduced *Carleson measures* to solve a problem in analytic interpolation. The same year, De Giorgi, and independently Nash, published proofs of regularity of solutions to elliptic and parabolic divergence form equations with bounded measurable coefficients. In 1961, John and Nirenberg's seminal paper appeared in CPAM, with their famous theorem about BMO functions. In the same issue of that journal, Moser's proof of elliptic regularity appeared. Then, in 1971, C. Fefferman made a deep and unexpected connection between harmonic function theory, Carleson measures, and BMO, opening the door to a theory of boundary value problems for solutions to the Laplacian and beyond. It is remarkable how many sharp results in the theory of elliptic/parabolic boundary value problems have relied on Carleson measure estimates.

Alexei Poltoratski

Title: From spectral gaps to causal depth

Abstract: I will talk about the classical gap and type problems in Fourier analysis which focus on functions and measures with gaps in the support of their Fourier transform. These historic problems studied by many famous mathematicians can be viewed in the context of the uncertainty principle in harmonic analysis. They have multiple connections and applications in

various areas of analysis and spectral theory. The second part of the title refers to recently found connections between the type problem and the theory of holographic horizons in high-energy physics.

Guillermo Rey Ley

Title: Merryfield's inequality for multiparameter martingales

Abstract: A classical result of Burkholder, Davis, and Gundy states that for martingales, the square function Sf and the maximal function f^* are comparable in L^p for $1 \leq p < \infty$. In the multiparameter setting, comparability for $p > 1$ follows by iteration, but the endpoint $p = 1$ requires new ideas. The biparameter case was established by Brossard.

   In this talk, we prove a martingale analogue of an inequality due to Merryfield, originally formulated in the continuous setting for harmonic functions on polyhalfspaces. As a consequence, we extend Brossard's L^1 estimate $\mathbb{E}[Sf] \lesssim \mathbb{E}[f^*]$ to an arbitrary number of parameters, for regular filtrations satisfying the Cairoli–Walsh (F_4) condition.

Oliver Roth

Title: Old and new on maximal Blaschke products

Abstract: Maximal Blaschke products were introduced in 2013 to better understand the case of equality in the Ahlfors–Nehari–Schwarz lemma. This talk aims to give an overview of the current state of affairs, with an emphasis on some new developments

Kristina Skreb

Title: Norm-variation of cubic ergodic averages

Abstract: We prove a quantitative result on norm convergence of cubic ergodic averages with respect to $d \geq 1$ commuting measure-preserving transformations. We use harmonic analysis techniques, a key tool being estimates for singular Brascamp-Lieb forms with cubical structure, which are used as a black box. The talk is based on joint work with Polona Durcik.

Alexander Volberg (Michigan State University, Ann Arbor MI, USA)

Title: Ground states of random d-local Hamiltonians

Abstract: Let $(C^2)^{\otimes n}$ denote a 2^n-dimensional complex Hilbert space, and consider $H(d,n) = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} g_j \Sigma_j$ a Hermitian operator, where g_j are standard independent gaussians and Σ_j are d qubits tensor products of n Pauli matrices, $d \leq n$. There are precisely $N = 3^d \binom{n}{d}$ such tensor products. It

is a classical problem in physics as well as in theoretical computer science to estimate the expectation of this random Hamiltonian, namely to estimate $\|H(d,n)\|_{op}$.

Different (d, n) regimes should give different asymptotics of this quantity when $n \to \infty$. We will give some new estimates when $d << n$ and will discuss the possible phase transitions. Hopefully I will be able to wave hands about relation to Parisi formula.