Universality, Zeta-Functions, and Chaotic Operators

August 7-11, 2023

One of the main goals of the workshop is to bring together the two largely independent communities investigating universality phenomena and related questions in order to present current progress and key research questions in their respective fields to each other. Some researchers continue the investigations of Voronin; this group can be considered as part of the analytic number theory community. The other group has its roots in the theory of functions and functional analysis. There exist several aspects of universality phenomena that are of interest in both communities, and an exchange of perspectives and ideas would definitely be fruitful and stimulating future research! In fact, there are already some successful joint works. Moreover, the upcoming 50th anniversary of Voronin's discovery would be a perfect occasion for this event.

The every day routine at Luminy is:

7:00-9:00 Breakfast

9:10-12:25 Morning Session

12:30-13:30 Lunch

14:30-17:45 Afternoon Session

19:30-20:30 Diner

Tuesday and Friday afternoon are free from lectures.

Monday

9:30-9:40 Welcome address

9:45-10:15	Karl Grosse-Erdmann	*Lévy's Phenomenon in the Plane and on the Disk*
10:20-10:40	BREAK	
10:40-11:10	Renata Macaitienė	*Limit Theorems for the Epstein Zeta-Function*
11:15-11:45	Athanasios Sourmelidis	*Continuous and Discrete Universality of Zeta-Functions: Two Sides of the Same Coin?*
11:50-12:20	Jürgen Müller	*Dynamics of the Backward Taylor Shift on Bergman Spaces*
12:30-13:30	LUNCH	
14:30-15:00	Vagia Vlachou	*Disjoint Universality Connected with Differential Operators*
15:05-15:35	Konstantinos Maronikolakis	*Properties of Abel Universal Functions*
15:40-16:00	BREAK	
16:00-16:30	Masahiro Mine	*On the Gonek Conjecture*
16:35-17:05	Roma Kačinskaitė	*One More Case of Discrete Universality for the Classes of Zeta-Functions*
17:10-17:40	Johan Andersson	*Universality with Scaling*

Chairperson: Kohji Matsumoto

Tuesday

9:10-9:40	Kohji Matsumoto	*On the Value-Distribution of the Logarithms of Symmetric Power L-functions in the Level Aspect*
9:45-10:15	Sumaia Saad Eddin	*Effective Universality for Dirichlet L-Functions with Respect to Characters*
10:20-10:40	BREAK	
10:40-11:10	Shota Inoue	*Joint Value Distribution of Dirichlet L-Functions in the Strip* $1/2 < \sigma < 1$
11:15-11:45	Lejla Smajlovic	*Study of the Ruelle Zeta Function Twisted by a Unitary Multiplier System at the Critical Point* $s = 0$ *and its Applications*
11:50-12:20	Darius Šiaučiūnas	*On Joint Universality of Dirichlet L-Functions*
12:30-13:30	LUNCH	

Chairperson: Jörn Steuding

The afternoon is free for excursions, hikes, small group math, whatever you want :-)

Wednesday

9:10-9:40	Romuald Ernst	*Furstenberg Families, Densities and Linear Dynamics*
9:45-10:15	Qeuntin Menet	*The Frequency of Chaos*
10:20-10:40	BREAK	
10:40-11:10	Anne-Maria Ernvall-Hytönen	*Euler's Divergent Series and Primes in Arithmetic Progressions*
11:15-11:45	Gautami Bhowmik	*Zeta-Functions of Lattices*
11:50-12:20	Ade Irma Suriajaya	*The Pair Correlation Conjecture, the Alternative Hypothesis, and an Unconditional Montgomery Theorem*
12:30-13:30	LUNCH	
14:30-15:00	Artūras Dubickas	*On the Fractional and Integral Parts of Geometric Progression*
15:05-15:35	Paulius Drungilas	*Multiplicative Dependence of Two Integers Shifted by a Root of Unity*
15:40-16:00	BREAK	
16:00-16:30	Ramūnas Garunkštis	*On Zeros of Zeta-Functions from the Extended Selberg Class*
16:35-17:05	Raivydas Šimėnas	*Hurwitz Zeta-Function is Prime*

Chairperson: Vagia Vlachou

Thursday

9:10-9:40	Masatoshi Suzuki	*Aspects of the Screw Function of the Riemann Zeta-Function Including Value-Distribution*
9:45-10:15	Łukasz Pańkowski	*Joint Value-Distribution of Shifts of the Riemann Zeta-Function*
10:20-10:40	BREAK	
10:40-11:10	Kenta Endo	*Effective Result for Multidimensional Denseness Theorem for L-Functions in the Selberg Class and its Application for Universality*
11:15-11:45	Keita Nakai	*Universality for the Iterated Integrals of Logarithms of L-Functions in the Selberg Class*
11:50-12:20	Neea Palojärvi	*Conditional Estimates for Logarithms and Logarithmic Derivatives in the Selberg Class*
12:30-13:30	LUNCH	
14:30-15:00	Saeree Wananiyakul	*Asymptotic Behavior of Sum of Values of Ideal Class Zeta-Function*
15:05-15:35	Tomas Kondratavičius	*Sum of the Epstein Zeta over the Riemann Zeta Zeros*
15:40-16:00	BREAK	
16:00-16:30	Marc Technau	*On Polynomials With Roots Modulo Almost All Primes*
16:35-17:?5	Problem Session	

Chairperson: Ramūnas Garunkštis

Friday

9:10-9:40	Myrto Manolaki	*On Universal Radial Approximation*
9:45-10:15	Vassili Nestoridis	*Universal Taylor Series in One Variable, on Open Questions and Approximation in Several Variables*
10:20-10:40	BREAK	
10:40-11:10	Christopher Hughes	*Discrete Moments of the Riemann Zeta Function*
11:15-11:45	Jörn Steuding	*Curves of the Riemann Zeta-Function and Universality*

11:50-12:00 Closing

Chairperson: Jörn Steuding

Code of Conduct

This code of conduct applies to all participants of our workshop.

All participants agree to be considerate in language and actions, respect the boundaries of fellow participants, and refrain from demeaning, discriminatory, or harassing behaviour and language.

We treat each other with respect and do not tolerate any forms of discrimination based on ethnicity, gender, disability, religion or belief, age or sexual orientation or identity. We believe that diversity provides new impulses, new ideas, new perspectives and innovation.

We conduct our research and support the research of others with integrity and according to the scientific standards. We are committed to scientific integrity and good scientific practice. We discuss problems and implement solutions openly and respectfully.

We recognize the achievements of others, give credit where it is necessary and offer constructive feedback. Science and innovation depend on an open discourse in which new ideas can be freely exchanged.

We are aware that research is often highly competitive and people can feel under pressure to produce results. Because of these circumstances, we are committed to being mindful of personal boundaries and contributing to a nurturing work environment based on trust and mutual respect.

— This code has been inspired by the one of the Max Planck Society —

Titles & Abstracts

in alphabetical order

Johan Andersson — Örebro University

Title: Universality with Scaling.

Abstract: We discuss recent work on universality of zeta and L-functions where in addition to imaginary shifts we allow scaling of its argument. In particular we discuss results close to the critical line where the Hurwitz zeta-function with a rational or transcendental parameter and with a suitable scaling of its argument may be modelled by certain stochastic integrals. As a consequence of these results we obtain new Universality theorems close to the critical line.

Gautami Bhowmik — Université de Lille

Title: Zeta Functions of Lattices.

Abstract: The analytic study of classical subgroup growth zeta functions of Grunewald, Segal and Smith was diversified by Petrogradsky (2007) using a multivariable version. We will present some recent applications of such functions to obtain information on the sublattices Λ of the free group $\mathbb{Z}^d$ such that $\mathbb{Z}^d/\Lambda$ has bounded rank. In particular the case where Λ is a polarised lattice is work in progress with Anirban Mukhopadhyay.

Paulius Drungilas — Vilnius University

Title: Multiplicative Dependence of Two Integers Shifted by a Root of Unity.

Abstract: We prove a result on the multiplicative independence of the numbers $m-\alpha$, $n-\alpha$, where $m>n$ are positive integers and α is a reciprocal algebraic number with the property that $\alpha+1/\alpha$ has at least two real conjugates over $\mathbb{Q}$ lying in the interval $(-\infty, 2]$. As an application, we show that for any positive integers $m>n$ and $k\geq 3$ the numbers $m-\zeta_k$, $n-\zeta_k$, where ζ_k is the primitive kth root of unity, are multiplicatively independent except when $(n,k)=(1,6)$. This settles a conjecture of Madritsch and Ziegler.

Artūras Dubickas — Vilnius University

Title: On the Fractional and Integral Parts of Geometric Progression

Abstract: Let $\xi \neq 0$ and $\alpha > 1$ be two real numbers. We review some results concerning the distribution of the sequence $\xi\alpha^n$, $n=0,1,2,\ldots$, which is an infinite geometric progression. The distribution of this sequence modulo 1 and the divisibility properties of its integral parts $\lfloor \xi\alpha^n \rfloor$ have been investigated for a long time. There is almost no progress in both problems for transcendental numbers α (except for the metrical results of Weyl (1916) and Koksma (1935)), so we will only consider algebraic numbers α. Although the sequence of fractional parts should be nicely (in fact, uniformly) distributed for 'most' ξ and α, its behaviour can be quite chaotic for some pairs ξ, α. In particular, some large regions of the interval $[0,1]$ can be free from the elements of this sequence at all. We are mainly interested in these two problems when α is a fixed rational number. Then, the main open question related to the sequence of fractional parts is that of Mahler (1968) when $\alpha=3/2$ and ξ is a positive real number. In the case of the sequence of integral parts $\lfloor \xi\alpha^n \rfloor$, $n=0,1,2,\ldots$, the divisibility questions become nontrivial already for rational integers $\alpha \geq 2$.

Kenta Endo — National Institute of Technology, Anan College Japan

Title: Effective Result for Multidimensional Denseness Theorem for L-Functions in the Selberg Class and its Application for Universality

*Abstract:*After the universality theorem for the Riemann zeta-function was proved by Voronin, he showed an effective multidimensional denseness theorem. In 2010, Garunkštis-Laurinčikas-Matsumoto-J. and R. Steuding used this theorem to obtain an "effective universality-type theorem". In this talk, we generalized these theorems to L-functions in the Selberg class.

Romuald Ernst — Université du Littoral Côte d'Opale

Title: Furstenberg Families, Densities and Linear Dynamics

Abstract: In this talk, I will be interested in giving an overview of some dynamical notions that appeared recently in Linear Dynamics arising from Furstenberg families. We will define the general notions and give some results that are specific to certain classes of Furstenberg families.

Anne-Maria Ernvall-Hytönen — University of Helsinki

Title: Euler's Divergent Series and Primes in Arithmetic Progressions

Abstract: Euler's divergent series $\sum_{n=0}^{\infty} n!z^n$ which converges only for $z = 0$ becomes an interesting object when evaluated with respect to a p-adic norm (which will be introduced in the talk). Very little is known about the values of the series. For example, it is an open question whether the value at one is irrational (or even non-zero). As individual values are difficult to reach, it makes sense to try to say something about collections of values over sufficiently large sets of primes. This leads to looking at primes in arithmetic progressions, which is in turn raises a need for an explicit bound

for the number of primes in an arithmetic progression under the generalized Riemann hypothesis.

During the talk, I will speak about both sides of the story: why we needed good explicit bounds for the number of primes in arithmetic progressions while working with questions about irrationality, and how we then proved such a bound. Later, we learned that Grenié and Molteni had already earlier derived a better bound for primes in arithmetic progressions assuming GRH. I will briefly speak about differences between the approaches.

The talk is joint work with Tapani Matala-aho, Neea Palojärvi and Louna Seppälä. (Questions about irrationality with T. M. and L. S. and primes in arithmetic progressions with N. P.).

RAMŪNAS GARUNKŠTIS — Vilnius University

Title: On Zeros of Zeta-Functions from the Extended Selberg Class

Abstract: Let $F(s)$ be a zeta-function from the extended Selberg class. In this talk we consider various relations between zeros of $F(s)$ and its derivative. For example, the Riemann zeta function and its derivative have the same number of zeros in small areas located to the left of the critical line and near it.

KARL GROSSE-ERDMANN — Université de Mons

Title: Lévy's Phenomenon in the Plane and on the Disk

Abstract: Lévy's phenomenon concerns the rate of growth of entire functions $f(z) = \sum_{n=0}^{\infty} a_n z^n$. According to the Wiman-Valiron theory,

$$\max_{|z|\leq r} |f(z)| \leq \mu(r)(\log \mu(r))^{1/2}$$

for any r outside a set of finite logarithmic measure, where $\delta > 0$ is a fixed number and μ denotes the maximum term of f. Erdős and Renyi

showed in 1969 that if one attributes random signs to the coefficients a_n then, almost surely, one can replace the factor $(\log \mu(r))^{1/2}$ by $(\log \mu(r))^{1/4}$. Such a phenomenon was first observed, in a special case, by Paul Lévy in 1930. We show that the same result holds if one randomizes the series as $f(z) = \sum_{n=0}^{\infty} a_n X_n z^n$, where $(X_n)_n$ is an i.i.d. sequence of centred subgaussian random variables. This answers positively a question of O. B. Skaskiv and improves on a partial answer by A. Kuryliak (2017). We also consider the corresponding question for holomorphic functions on the unit disk. The results have an application to the dynamics of operators on the spaces $H(\mathbb{C})$ and $H(\mathbb{D})$. This is joint work with Kevin Agneessens.

CHRISTOPHER HUGHES — University of York

Title: Discrete Moments of the Riemann Zeta Function

Abstract: Shanks conjectured that the mean of the derivative of the Riemann zeta function evaluated at the zeros of zeta is real and positive. This was proven in the 1980s by Conrey and Ghosh. We will talk about various extensions of this to higher derivatives and higher moments, giving a mix of heuristic and theoretical results. This work is joint with Andrew Pearce-Crump. .

SHOTA INOUE — Tokyo Institute of Technology

Title: Joint Value Distribution of Dirichlet L-Functions in the Strip $1/2 < \sigma < 1$

Abstract: The speaker will discuss the joint value distribution of Dirichlet L-functions in the strip $1/2 < \sigma < 1$. He will present results for dependence property and joint extreme values of Dirichlet L-functions.

Roma Kačinskaitė — Vilnius University

Title: One More Case of Discrete Universality for the Classes of Zeta-Functions

Abstract: The first result related to the mixed discrete joint universality theorem on approximation of certain target couple of analytic functions by the shifts of a pair consisting of the function $\varphi(s)$ belonging to wide class of Matsumoto zeta-functions and the periodic Hurwitz zeta-function $\zeta(s,\alpha;\mathfrak{B})$ was obtained in [1]. This was done under a condition that the elements of the set $\big\{(\log p : p \in \mathbb{P}), (\log(m+\alpha) : m \in \mathbb{N}\cup\{0\}), \frac{2\pi}{h}\big\}$ are linearly independent over $\mathbb{Q}$. Later universality for the class of partial zeta-functions $\varphi_h(s)$ and $\zeta(s,\alpha;\mathfrak{B})$ was considered in the case of rational number $\exp\{\frac{2\pi}{h}\}$ and transcendental number α (see [2]). In the talk, we present a new result of mixed discrete joint universality theorem for the tuple $\big(\varphi(s), \zeta(s,\alpha;\mathfrak{B})\big)$, where instead of the class of partial zeta-functions $\varphi_h(s)$ the class of the original functions $\varphi(s)$ itself is studied but under conditions to be $\exp\{\frac{2\pi}{h}\}$ rational number and parameter α transcendental. This is a joint work by R. Kačinskaitė, K. Matsumoto and Ł. Pańkowski (see [3]). It is necessary to mention that in general the arithmetic nature of the number h plays a crucial role in the proof of discrete universality type theorems.

References

[1] R. Kačinskaitė, K. Matsumoto. On mixed joint discrete universality for a class of zeta-functions. In: *Proc. of 6th Intern. Conf., Palanga, Lithuania, 2016,* Anal. Probab. Methods Number Theory, A. Dubickas et al. (Eds.), Vilnius: Vilnius University Publ. House, pp. 51–66 (2017).

[2] R. Kačinskaitė, K. Matsumoto. The discrete case of the mixed joint universality for a class of certain partial zeta-functions. *Taiwanese J.*

Math., **25**(4), 647–663 (2021).

[3] R. Kačinskaitė, K. Matsumoto, Ł. Pańkowski. On mixed joint discrete universality for a class of zeta-functions: one more case. *Taiwanese J. Math.*, **27**(2), 221–236 (2023).

TOMAS KONDRATAVIČIUS — Vilnius University

Title: Sum of the Epstein zeta over the Riemann Zeta Zeros

Abstract: We investigate the sum of values of the Epstein zeta function $Z(s,\alpha)$ at non-trivial zeros of the Riemann zeta function $\zeta(s)$. .

RENATA MACAITIENĖ — Šiauliai Academy, Vilnius University

Title: Limit Theorems for the Epstein Zeta-Function

Abstract: In the talk, some results of joint works with A. Laurinčikas on the value-distribution of the Epstein zeta-function will be presented. That is, the generalized Bohr-Jessen type limit theorems in the sense of the weak convergence of probability measures on the complex plane will be considered, and the explicit forms of the limit measures will be given.

Let Q be a positive definite quadratic $n \times n$ matrix and $Q[\underline{x}] = \underline{x}^{\mathrm{T}} Q \underline{x}$ for $\underline{x} \in \mathbb{Z}^n$. The Epstein zeta-function $\zeta(s;Q)$, $s = \sigma + it$, is defined, for $\sigma > \frac{n}{2}$, by the series

$$\zeta(s;Q) = \sum_{\underline{x} \in \mathbb{Z}^n \setminus \{\underline{0}\}} (Q[\underline{x}])^{-s},$$

and can be continued analytically to the whole complex plane, except for a simple pole at the point $s = \frac{n}{2}$ with residue $\frac{\pi^{n/2}}{\Gamma(n/2)\sqrt{\det Q}}$. The function $\zeta(s;Q)$ was introduced by P. Epstein in [1], its value-distribution was investigated by various authors; for example, an extensive survey of the results for the function $\zeta(s;Q)$ is given in [2].

We will present the probabilistic limit theorems of continuous and discrete types for $\zeta(s;Q)$ with even $n \geq 4$ and integers $Q[\underline{x}]$. More precisely, it will be discussed that, for a special differentiable function $\varphi(t)$ with a monotonic derivative,

$$\frac{1}{T}\text{meas}\left\{t \in [0,T] : \zeta(\sigma + i\varphi(t);Q) \in A\right\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

converges weakly to an explicitly given probability measure on $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$ as $T \to \infty$ [3]. Here meas A denotes the Lebesgue measure of a measurable set $A \subset \mathbb{R}$, and $\mathcal{B}(\mathbb{C})$ – the Borel σ-field of the space $\mathbb{C}$. Furthermore, it will be showed that, for a certain increasing differentiable function $\varphi(t)$ with a continuous monotonic bounded derivative and with an additional condition for the sequence $\{\varphi(k)\}$, on $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$, there exists an explicitly described probability measure $P_{Q,\sigma}$ such that

$$\frac{1}{N}\#\left\{N \leqslant k \leqslant 2N : \zeta(\sigma + i\varphi(k);Q) \in A\right\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

converges weakly to $P_{Q,\sigma}$ as $N \to \infty$ [4]. Also, a few examples of the function $\varphi(t)$ will be given.

References

[1] Epstein, P. Zur Theorie allgemeiner Zetafunktionen, *Math. Ann.,* **56**, 615–644, 1903.

[2] Nakamura, T., Pańkowski, Ł. On zeros and c-values of Epstein zeta-functions, *Šiauliai Math. Semin.,* **8**(16), 181–195, 2013.

[3] Laurinčikas, A., Macaitienė, R. A generalized Bohr–Jessen type theorem for the Epstein zeta-function. *Mathematics,* 10(12): 2042, 1–11, 2022.

[4] Laurinčikas, A., Macaitienė, R. A generalized discrete Bohr–Jessen type theorem for the Epstein zeta-function. *Mathematics,* 11(4): 799, 1–13, 2023.

Myrto Manolaki — University College Dublin)

Title: On Universal Radial Approximation

Abstract: Generally speaking, a function is called universal if, via a countable process, it can approximate a maximal class of functions. This talk is concerned with the class of Abel universal functions which consists of all holomorphic functions on the unit disc whose radial limits can uniformly approximate all continuous functions on proper compact subsets of the unit circle. We will discuss a wide range of properties of Abel universal functions and make comparisons with the corresponding properties of functions with universal Taylor series. Joint work with Stéphane Charpentier and Konstantinos Maronikolakis.

Konstantinos Maronikolakis — University College Dublin

Title: Properties of Abel Universal Functions

Abstract: I will focus on the class of Abel universal functions which are holomorphic functions on the unit disk whose radial limits uniformly approximate all possible continuous functions on compact subsets of the unit circle. More precisely, given an increasing sequence $\rho = (r_n)_n \in [0,1)$ tending to 1, a holomorphic function f on the open unit disk is an Abel universal function (with respect to ρ) if for any compact set K on the unit circle, different from the unit circle, the set of functions $\{f(r_n\cdot)|K : n \in N\}$ is dense in the space of continuous functions on K. I will discuss properties of Abel universal functions and also give connections to the classical subspaces of holomorphic functions on the unit disk (Hardy, Bergman, Dirichlet spaces). Joint work with Stéphane Charpentier and Myrto Manolaki.

Kohji Matsumoto — Nagoya University

Title: On the Value-Distribution of the Logarithms of Symmetric Power L-Functions in the Level Aspect

Abstract: The aim of this presentation is to report some new results on the value-distribution of symmetric power L-functions, analogous to the theory of M-functions associated to Dirichlet L-functions due to Ihara and the speaker. In particular, in the symmetric square case, we can show a limit theorem for rather wide class of test functions (under the assumption of the GRH). We also present some attempt to more general symmetric power case. This ia a joint work with Philippe Lebacque, Masahiro Mine and Yumiko Umegaki.

Quentin Menet — Université de Mons

Title: The Frequency of Chaos

Abstract: In linear dynamics, an operator T is said to be hypercyclic when it possesses a dense orbit, i.e. a vector x such that the sets $N(x, U) = \{n \geq 1 : T^n x \in U\}$ is non-empty (or equivalenty infinite) for every non-empty open set U. During the last two decades, researchers tried to better understand when we can require that there is an orbit visiting each non-empty open set by respecting some frequency. Several stronger notions of hypercyclicity have then been introduced (frequent hypercyclicity, U-frequent hypercyclicity, reiterative hypercyclicity,...) depending on the considered frequency. Another interesting notion in linear dynamics is the notion of chaotic operators, i.e hypercyclic operators with a dense set of periodic points. A natural question is “Which impact the existence of a dense set of periodic points can have on the frequency of visits of some dense orbits?”. In this talk, we will try to give an overview on the main results related to this question.

Masahiro Mine — Sophia University

Title: On the Gonek Conjecture

Abstract: Universality of the Hurwitz zeta-function is one of the central topics in the theory of zeta-functions. First, it was proved that the Hurwitz zeta-function has universality if the parameter is rational or transcendental. Gonek conjectured in his thesis that the result remains true even if the parameter is algebraic irrational. This conjecture is still open, although some progress has been made by several mathematicians. The difficulty seems to lie in the complexity of the random model for the Hurwitz zeta-function with algebraic irrational parameter. The main result of this talk is a weak form of universality of the Hurwitz zeta-function with algebraic irrational parameter. It provides significant progress on Gonek's conjecture. The key idea for the proof is using two random Dirichlet series related to the Hurwitz zeta-function, one of which is easy to deal with. Furthermore, we apply the result to show that many of such Hurwitz zeta-functions have infinitely many zeros in the right half of the critical strip.

Jürgen Müller — University of Trier

Title: Dynamics of the Backward Taylor Shift on Bergman Spaces

Abstract: For $p \geq 1$ and open sets Ω in the extended complex plane containing the origin, the backward Taylor shift on the Bergman space $A^p(\Omega)$ is defined by

$$(Tf)(z) := \begin{cases} (f(z) - f(0))/z, & z \neq 0 \\ f'(0), & z = 0 \end{cases}.$$

Depending on Ω and p, the operator shows a rich variety concerning its dynamical behaviour. Some aspects are discussed in the talk.

Keita Nakai — Nagoya University

Title: Universality for the Iterated Integrals of Logarithms of L-Functions in the Selberg Class

Abstract: The denseness of the Riemann zeta-function has been studied for a long time. However, the problem that the set of the Riemann zeta-function at critical line is dense or not in the complex plane is still open. For this reason, Endo and Inoue considered the iterated integrals of logarithms of the Riemann zeta-function and obtained a new result relating this denseness problem and in 2022, Endo proved the universality theorem for this function. In this talk, we give a more generalization of the universality theorem for the iterated integrals of logarithms of L-functions in the Selberg class.

Vassili Nestoridis — National and Kapodistrian University of Athens

Title: Universal Taylor Series in One Variable, on Open Questions and Approximation in Several Variables

Abstract: The existence of universal Taylor series implies that, if a power series f overconverges towards g, the function g is not necessarily a continuation of f. It is also open if g can also be defined on an open set in the domain of holomorphy of f. Extensions to the case of several variables are also possible.

Neea Palojärvi — University of Helsinki

Title: Conditional Estimates for Logarithms and Logarithmic Derivatives in the Selberg Class

Abstract: The Selberg class consists of functions sharing similar properties to the Riemann zeta function. The Riemann zeta function is one example

of the functions in this class. The estimates for logarithms of Selberg class functions and their logarithmic derivatives are connected to, for example, primes in arithmetic progressions.

In this talk, I will discuss about effective and explicit estimates for logarithms and logarithmic derivatives of the Selberg class functions when $\Re(s) \geq 1/2 + \delta$ where $\delta > 0$. All results are under the Generalized Riemann hypothesis and some of them are also under assumption of a polynomial Euler product representation or the strong λ-conjecture. The talk is based on a joint work with Aleksander Simonič (University of New South Wales Canberra).

Łukasz Pańkowski — Adam Mickiewicz University, Poznań

Title: Joint Value-Distribution of Shifts of the Riemann Zeta-Function

Abstract: We prove that any complex values $z_1, ..., z_n$ can be approximated by the following shifts of the Riemann zeta-function $\zeta(s + id_1t), ..., \zeta(s + id_nt)$ for infinitely many t, provided $d_1, ..., d_n$ are distinct and positive rational numbers, and s is a fixed complex number lying in the right open half of the critical strip.

Sumaia Saad Eddin — Austrian Academy of Sciences

Title: Effective Universality for Dirichlet L-functions with respect to Characters

Abstract: In 1975, Voronin proved his famous universality theorem for the zeta-function, which states that any non-vanishing analytic function can be approximated uniformly by certain purely imaginary shifts of the zeta-function in the critical strip. This stunning approximation property is called universality. It has many critical applications to the theory of distribution of values, the Riemann hypothesis, algebraic number theory, and physics.

Voronin's theorem attracted many mathematicians, who have had exciting contributions to the universality theory development by improving and extending it in various directions. Unfortunately, the known proofs of universality theorems are ineffective. For this reason, one may say that effective universality is a big challenge in universality theory. In this talk, we review some important results in this regard. In particular, we talk about the universality theorems due to Bagchi and Eminyan, concerned with a character satisfying certain approximation conditions.

Raivydas Šimėnas — Vilnius University

Title: Hurwitz Zeta Function is Prime

Abstract: By analogy with natural numbers, there is a notion of factorization for meromorphic functions in terms of functional composition. Suppose $F(s) = g(h(s))$ with F meromorphic. In this expression, g must be meromorphic and h entire, or g must be rational and h meromorphic. This is called a *decomposition*. If g or h in a decomposition must be linear, F is *prime*. We show that the Hurwitz zeta function is prime.

Lejla Smajlovic — University of Sarajevo

Title: Study of the Ruelle Zeta Function Twisted by a Unitary Multiplier System at the Critical Point $s = 0$ and its Applications

Abstract: We study the Ruelle zeta function associated to cofinite hyperbolic Riemann surfaces with ramification points twisted by a unitary multiplier system of arbitrary real, admissible weight $2k$. Using the properties of the zeta-regularized determinants we derive a new form of a functional equation for this function and compute explicitly the first term in the Laurent or Taylor series expansion of this function at $s = 0$. This term has an important connection to the Reidemeister torsion when the surface is cocompact and is

related to special values of L-functions when the surface is arithmetic. We present three applications. The first two are related to the higher-dimensional Rademeister torsion for Seifert fibered spaces with acyclic n-dimensional representations. The last application is related to evaluation of the lead term at $s = 0$ in terms of special values of Dirichlet L-functions, when the underlying Fuchsian group is the congruence subgroup of the modular group. This work is joint with Jay Jorgenson and Min Lee.

Jörn Steuding — University of Würzburg

Title: Curves of the Riemann Zeta-Function and Universality

Abstract: In 2015, Gonek and Montgomery, building on work of Yildirim, proved that the curve generated by the values of the zeta-function on the critical line $1/2 + i\mathbb{R}$ turns in clockwise direction for all sufficiently large imaginary parts subject to the truth of the Riemann Hypothesis. A straightforward application of Voronin's universality theorem shows that the behaviour on vertical lines to the right, i.e., $a + i\mathbb{R}$ with $1/2 < a < 1$, is rather different. Extending this observation, we demonstrate that every such zeta-function curve resulting from a vertical line in the open right half of the critical strip contains every smooth plane curve (and even curiosities as the Peano curve) up to an arbitrarily small error. Finally, we discuss in which cases the image of such curves lies dense in the complex plane. This is joint work with Athanasios Sourmelidis (TU Graz).

Athanasios Sourmelidis — Graz University of Technology

Title: Continuous and Discrete Universality of Zeta-Functions: Two Sides of the Same Coin?

Abstract: In 1975 Voronin proved his celebrated universality theorem for the Riemann zeta-function $\zeta(s)$ which roughly says that for any admissible function $f(s)$ there is $\tau > 0$ such that $\zeta(s + i\tau) \approx f(s)$ in its domain of definition. If τ can be taken only by a discrete subset A of the real line, then we talk about discrete universality, otherwise we have continuous universality. The proofs of these two concepts are almost identical but it is not known whether the one implies the other. We will discuss this and related questions. In particular, if A is an arithmetic progression then there is indeed a link between the continuous and the discrete case which is easier to perceive if we translate the question in the language of linear dynamics.

Ade Irma Suriajaya — Kyushu University

Title: The Pair Correlation Conjecture, the Alternative Hypothesis, and an Unconditional Montgomery Theorem

Abstract: Assuming the Riemann Hypothesis (RH), Montgomery (1973) proved a theorem concerning the pair correlation of nontrivial zeros of the Riemann zeta-function. One consequence of this theorem was that, under RH, at least 2/3 of the zeros are simple. We show that this theorem of Montgomery holds unconditionally. Furthermore, under a much weaker hypothesis than RH, we show that at least 61.7% of zeros of the Riemann zeta-function are simple. This weaker hypothesis does not require that any of the zeros are on the half-line. In this talk, we will also discuss how to improve this result. Montgomery's theorem states an asymptotic behavior of a function $F(\alpha)$ which captures the pair correlation of nontrivial zeros of the Riemann zeta-function in the interval [-1,1] and he gave the now famous "pair correla-

tion conjecture" predicting the behavior of $F(\alpha)$ beyond this integral. This conjecture, also assuming RH, implies that almost all zeros of the Riemann zeta-function are simple. An alternative to Montgomery's conjecture, known as the Alternative Hypothesis (AH) is an antithetical statement that consecutive zeros of the Riemann zeta-function are spaced at multiples of half of their average spacing. This hypothesis arose in the mid 1990s and has been studied by several authors, including Conrey and Iwaniec (2002), Farmer, Gonek and Lee (2014), Baluyot (2016), Lagarias and Rodgers (2020). We examine further consequences of AH and show that, under RH, AH seems to allow for a certain percentage of multiple zeros. We also prove that a slightly stronger version of AH implies that almost all zeros of the Riemann zeta-function are simple. This is joint work with Siegfred Alan C. Baluyot, Daniel Alan Goldston, and Caroline L. Turnage-Butterbaugh.

MASATOSHI SUZUKI — Tokyo Institute of Technology

Title: Aspects of the Screw Function of the Riemann Zeta-Function Including Value-Distribution

Abstract: The screw function in the title is a classical object in analysis and is closely related to positive definite integral kernels. In this talk, we will introduce the screw function of the Riemann zeta-function and describe some relations between its analytic properties and the distribution of the zeros of the Riemann zeta-function. As an application, Weil's criterion for the RH is reformulated in terms of functions rather than distributions. We also plan to mention the value distribution of the screw function of the Riemann zeta-function.

Darius Šiaučūnas — Šiauliai Academy, Vilnius University

Title: On Joint Universality of Dirichlet L-Functions

Abstract: Let $s = \sigma + it$ be a complex variable and χ be a Dirichlet character modulo $q \in \mathbb{N}$. The Dirichlet L-function $L(s, \chi)$ is defined, for $\sigma > 1$, by

$$L(s,\chi) = \sum_{m=1}^{\infty} \frac{\chi(m)}{m^s} = \prod_p \left(1 - \frac{\chi(p)}{p^s}\right)^{-1},$$

where the product is taken over all prime numbers. If $\chi(m)$ is the principal character, then

$$L(s,\chi) = \zeta(s) \prod_{p|q} \left(1 - \frac{1}{p^s}\right)^{-1},$$

where $\zeta(s)$ is the Riemann zeta-function. Therefore, in this case, the function $L(s, \chi)$ has analytic continuation to the whole complex plane, except for the point $s = 1$ which is a simple pole with residue $\prod_{p|q}(1 - 1/p)$. If $\chi(m)$ is a non-principal character, then $L(s, \chi)$ has analytic continuation to an entire function. Since 1975, it is known [3] that the Dirichlet L-functiom has an universality property. Moreover, for Dirichlet L-functions, a more complicated kind of universality, the joint universality, is considered. The first result in this direction also belongs to S.M. Voronin. In [4], investigating the functional independence of Dirichlet L-functions, he obtained in a not entirely explicit form the joint universality of these functions. For the modern version of a joint universality theorem, we need some notation. Denote by $\mathcal{K}$ the class of compact subsets of the strip D with connected complements, and by $H_0(K)$ with $K \in \mathcal{K}$ the class of continuous non-vanishing functions on K that are analytic in the interior of K. Let $\operatorname{meas} A$ be the Lebesgue measure of a measurable set $A \subset \mathbb{R}$. A modern version of Voronin joint universality theorem, see, for example, [1], is the following theorem.

Theorem 1 *Suppose that $\chi_1, \dots, \chi_r$ are pairwise non-equivalent Dirichlet characters. For $j = 1, \dots, r$, let $K_j \in \mathcal{K}$ and $f_j(s) \in H_0(K)$. Then, for*

every $\varepsilon > 0$*,*

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas} \left\{ \tau \in [0,T] : \sup_{1\le j\le r} \sup_{s\in K_j} |L(s+i\tau,\chi_j) - f_j(s)| < \varepsilon \right\} > 0.$$

In joint universality theorems, the shifts $L(s+i\tau,\chi_j)$, $j=1,\dots,r$, in some sense must be independent. For this, in Theorem 1, the pairwise non-equivalence of Dirichlet characters was applied. Ł. Pańkowski, in [2], used the generalized shifts $(L(s+i\alpha_1\tau^{a_1}\log^{b_1}\tau,\chi_1),\dots,L(s+i\alpha_r\tau^{a_r}\log^{b_r}\tau,\chi_r))$. Our report is devoted to approximation of analytic functions by more general shifts. Suppose that, for $j=1,\dots,r$, $\gamma_j(\tau)$ is an increasing real continuously differentiable function on $[T_0,\infty)$ with monotonic derivative $\gamma_j'(\tau) = \delta_j(\tau)(1+o(1))$ such that $\delta_1(\tau) = o(\delta_2(\tau))$, $\dots \delta_{r-1}(\tau) = o(\delta_r(\tau))$ and

$$\gamma_j(2\tau) \max_{\tau\leqslant u\leqslant 2\tau} (\gamma_j'(u))^{-1} \ll \tau.$$

Theorem 2 *Suppose that* $\gamma_1(\tau),\dots,\gamma_r(\tau)$ *satisfy the above hypotheses. For* $j=1,\dots,r$*, let* $K_j \in \mathcal{K}$ *and* $f_j \in H_0(K_j)$*. Then, for every* $\varepsilon > 0$*,*

$$\liminf_{T\to\infty} \frac{1}{T-T_0} \mathrm{meas} \left\{ \tau \in [T_0,T] : \sup_{1\leqslant j\leqslant r} \sup_{s\in K_j} |L(s+i\gamma_j(\tau),\chi_j) - f_j(s)| < \varepsilon \right\} > 0.$$

Moreover, "$\liminf$*" can be replaced by "*$\lim$*" for all but at most countably many* $\varepsilon > 0$*.*

Moreover, the joint universality of Dirichlet L-functions with shifts $(L(s+it_\tau^{\alpha_1},\chi_1),\dots,L(s+it_\tau^{\alpha_r},\chi_r))$, where t_τ is the Gram function and $\alpha_1,\dots,\alpha_r$ are fixed different positive numbers, can be obtained. Generalizations for some compositions $F(L(s,\chi_1),\dots,L(s,\chi_r))$ are possible as well.

References

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MARC TECHNAU — Graz Technical University

Title: On Polynomials With Roots Modulo Almost All Primes

Abstract: Call a monic integer polynomial *exceptional* if it has a root modulo all but a finite number of primes, but does not have an integer root. We construct exceptional polynomials with all factors of the form $X^p - b$, p prime and b square free. Time permitting, we shall also discuss the problem of completing an irreducible monic integer polynomial towards being exceptional by means of multiplication by a quadratic factor. This is joint work with Christian Elsholtz and Benjamin Klahn.

VAGIA VLACHOU — University of Patras

Title: Disjoint Universality Connected with Differential Operators

Abstract: For a simply connected domain G, we study the problem of disjoint universality for the sequences of operators $T_{\alpha,n} : H(G) \to H(G)$, defined by $T_{\alpha,n}(f)(z) = \sum_{j=0}^{n} \frac{f^{(j)}(z)}{j!}(\alpha z)^j$, where $\alpha \in \mathbb{C} \setminus \{0\}$. Note that $T_{\alpha,n}(f)(z)$ is the n^{th} partial sum of the Taylor expansion of f around z on $(\alpha+1)z$. The motivation to study such sequences comes from universal Taylor series, by changing the role of the center of expansion.

Saeree Wananiyakul — Chulalongkorn University

Title: Asymptotic Behavior of Sum of Values of Ideal Class Zeta-Function

Abstract: We prove the upper bound for the sum of values of the ideal class zeta function over nontrivial zeros of the Riemann zeta-function. The same result for the Dedekind zeta-function is also obtained. This may shed light on some unproven cases of the Dedekind conjecture in general.

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