



Einladung zum Oberseminar Mathematik in den Naturwissenschaften

Julius-Maximilians-Universität Würzburg
Lehrstuhl für Mathematik in den Naturwissenschaften

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Symmetry breaking for local/nonlocal interaction functionals in diffuse formulation

In this talk we will consider a diffuse interface generalized antiferromagnetic model. This model can be described by the following mean field free energy functional. For $L, J, \varepsilon > 0$, $d \geq 1$, $p \geq d + 2$, $u \in W_{loc}^{1,2}(\mathbb{R}^d; [0, 1])$ and $[0, L]^d$ -periodic, define

$$\begin{aligned} \tilde{\mathcal{F}}_{J,L,\varepsilon}(u) := & \frac{J}{L^d} \left[3\varepsilon \int_{[0,L]^d} \|\nabla u(x)\|_1^2 dx + \frac{3}{\varepsilon} \int_{[0,L]^d} W(u(x)) dx \right] - \\ & \frac{1}{L^d} \int_{\mathbb{R}^d} \int_{[0,L]^d} |u(x+\zeta) - u(x)|^2 K(\zeta) dx d\zeta, \end{aligned} \quad (1)$$

where, for $y = (y_1, \dots, y_d) \in \mathbb{R}^d$, $\|y\|_1 = \sum_{i=1}^d |y_i|$, $W(t) = t^2(1-t)^2$ and $K(\zeta) = \frac{1}{(\|\zeta\|_1+1)^p}$.

This type of local/nonlocal interaction functionals, with suitable choices of the kernel K , is used to model pattern formation in several contexts, among which thin-magnetic films ($p = d + 1$), diblock copolymer melts ($p = d - 2$) and colloidal systems ($p = d - 2$ and exponential decay). Periodic patterns in the ground states are expected to emerge by the competition between the first term, short-range and attractive, and the second term, long-range and repulsive. Depending on the mutual strength between the two terms, modulated in this case by the constant J , different patterns are expected to occur. In any dimension $d \geq 1$ for a suitable range of J and for sufficiently small $\varepsilon > 0$, it is conjectured that the minimizers are non-constant one-dimensional periodic functions. In particular, minimizers display, in dimension $d \geq 2$, less symmetries than the functional itself. In the literature this phenomenon is called symmetry breaking. Our result, obtained in collaboration with S. Daneri and E. Runa, consists in proving this conjecture, namely that in general dimension and for a suitable range of J the minimizers of (1) are one dimensional periodic functions (and thus symmetry breaking occurs).

Ort: Zoom video conference

Zeit: Donnerstag, 10.12.2020 um 14:15 Uhr

You are cordially invited to this lecture. Request the Zoom link from
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gez. Anja Schlömerkemper