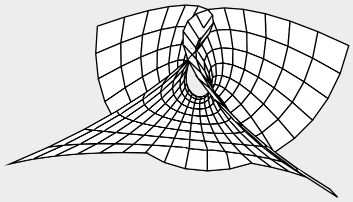

Einladung zum Oberseminar Mathematik in den Naturwissenschaften

Julius-Maximilians-Universität Würzburg
Lehrstuhl für Mathematik in den Naturwissenschaften

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On an electrostatic problem and a new class of exceptional subdomains of $\mathbb{R}^3$

In potential theory, a smooth domain Ω of the Euclidean space $\mathbb{R}^N$ is called exceptional if there exists a harmonic function u in Ω which satisfies the overdetermined boundary conditions

$$u|_{\partial\Omega} = 0 \qquad \partial_\nu u|_{\partial\Omega} = C \in \mathbb{R},$$

where ∂_ν denotes the outer normal derivative on $\partial\Omega$. Half spaces in $\mathbb{R}^N, N \geq 1$ and complements of balls in $\mathbb{R}^N, N \geq 2$ are trivial examples of exceptional domains.

The problem of finding exceptional domains was first studied by L. Hauswirth, F. Hélein, and F. Pacard in the seminal paper [1] and the classification of planar expectional domains characterized by additional assumptions was obtained by Khavinson, Lundberg and Teodorescu [2], and independently by Traizet [4]. Despite the significant efforts in the previous literature, the structure of the set of exceptional domains in dimensions $N \geq 3$ remains largely unknown.

In this talk, we present a construction (via bifurcation theory) of a new class of exceptional subdomains of the form

$$\Omega_\phi := \{(z,t) \in \mathbb{R}^2 \times \mathbb{R} : |z| \geq \phi(t)\} \subset \mathbb{R}^3,$$

where $\phi : \mathbb{R} \to (0,\infty)$ is a 2π-periodic function of class $\mathcal{C}^{2,\alpha}$ for some $\alpha \in (0,1)$.

The domains we construct solve an electrostatic problem, as the constant charge distribution on the surface $\partial\Omega_\phi$ is an electrostatic equilibrium. Among bounded regular surfaces S, only the round sphere has this property by results of Mendez and Reichel [3, 5] confirming a conjecture of P. Gruber. Our result is the first to prove the existence of nontrivial unbounded surfaces $S \subset \mathbb{R}^3$ enjoying the same property.

References

[1] L. Hauswirth, F. Laurent, Hélein, F. Pacard, *On an overdetermined elliptic problem.* Pacific J. Math. 250 (2011), no. 2, 319-334.

[2] D. Khavinson, E. Lundberg and R. Teodorescu, *An overdetermined problem in potential theory* Pacific Journal of Mathematics, 2013, vol 265, 85-1

[3] O. Mendez and W. Reichel, *Electrostatic characterization of spheres*(2000): 223-245.

[4] M. Traizet, *Classification of the solutions to an overdetermined elliptic problem in the plane.* Geom. Funct. Anal. 24 (2014), 690-720.

[5] W. Reichel, *adial symmetry for elliptic boundary-value problems on exterior domains*(Arch. Rational Mech. Anal. 137 (1997). 381-394

Ort: Mathmatik Ost, 40.03.003	Zeit: Dienstag, 05.07.2022 um 10:00 Uhr

You are cordially invited to this lecture.

gez. Anja Schlömerkemper