On an electrostatic problem and a new class of exceptional subdomains of $\mathbb{R}^3$

In potential theory, a smooth domain $\Omega$ of the Euclidean space $\mathbb{R}^N$ is called exceptional if there exists a harmonic function $u$ in $\Omega$ which satisfies the overdetermined boundary conditions

$$u|_{\partial\Omega} = 0 \quad \partial_{\nu} u|_{\partial\Omega} = C \in \mathbb{R},$$

where $\partial_{\nu}$ denotes the outer normal derivative on $\partial\Omega$. Half spaces in $\mathbb{R}^N, N \geq 1$ and complements of balls in $\mathbb{R}^N, N \geq 2$ are trivial examples of exceptional domains.

The problem of finding exceptional domains was first studied by L. Hauswirth, F. Hélein, and F. Pacard in the seminal paper [1] and the classification of planar exceptional domains characterized by additional assumptions was obtained by Khavinson, Lundberg and Teodorescu [2], and independently by Traizet [4]. Despite the significant efforts in the previous literature, the structure of the set of exceptional domains in dimensions $N \geq 3$ remains largely unknown.

In this talk, we present a construction (via bifurcation theory) of a new class of exceptional subdomains of the form

$$\Omega_{\phi} := \{ (z, t) \in \mathbb{R}^2 \times \mathbb{R} : |z| \geq \phi(t) \} \subset \mathbb{R}^3,$$

where $\phi : \mathbb{R} \rightarrow (0, \infty)$ is a $2\pi$-periodic function of class $C^{2,\alpha}$ for some $\alpha \in (0, 1)$.

The domains we construct solve an electrostatic problem, as the constant charge distribution on the surface $\partial\Omega_{\phi}$ is an electrostatic equilibrium. Among bounded regular surfaces $S$, only the round sphere has this property by results of Mendez and Reichel [3, 5] confirming a conjecture of P. Gruber. Our result is the first to prove the existence of nontrivial unbounded surfaces $S \subset \mathbb{R}^3$ enjoying the same property.

References


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You are cordially invited to this lecture.

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