The fractional Yamabe problem with singularities

The so-called Yamabe problem in conformal geometry asks for a metric conformal to a given one with constant scalar curvature. From the analytic point of view, this problem becomes a semilinear elliptic PDE with critical (for the Sobolev embedding) power non-linearity. If we study the problem in the Euclidean space, allowing the presence of nonzero-dimensional singularities (given by smooth submanifolds) can be transformed into reducing the non-linearity to a Sobolev-subcritical power. In this case, the work of Schoen and Yau shows that to obtain a complete metric, the singular set must satisfy a dimensional restriction. Under this assumption, singular solutions exist and have been constructed. A quite recent notion of non-local curvature gives rise to a parallel study which weakens the geometric assumptions of positive scalar curvature giving rise to a non-local semilinear elliptic (Sobolev-critical) PDE.

In this talk, we will focus on metrics that are singular along nonzero-dimensional singularities. In collaboration with Ao, Chan, Fontelos, González and Wei, we covered the construction of solutions that are singular along (zero and positive dimensional) smooth submanifolds in this fractional setting. This was done through the development of new methods coming from conformal geometry and scattering theory for the study of non-local ODEs. Due to the limitations of the techniques we used, the particular case of maximal possible dimension for the singularity was not covered. In a recent work, in collaboration with H. Chan, we cover this specific dimension constructing and studying singular solutions of critical dimension.

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You are cordially invited to this lecture.