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The hybrid phase field models

We have developed two new phase field models, which are given by

$$S_t(t,x) = -c\left(\partial_S\psi\big(S(t,x)\big) - \nu\Delta_x S(t,x)\right)|\nabla_x S(t,x)|,$$

$$S_t(t,x) = c\,\mathrm{div}_x\Big(\nabla_x\big(\partial_S\psi(S(t,x)) - \nu\Delta_x S(t,x)\big)|\nabla_x S(t,x)|\Big).$$

Here $S(t,x) \in \mathbb{R}$ is the order parameter and ψ is a double well potential. Formally, these evolution equations differ from the Allen-Cahn equation and the Cahn-Hilliard equation by the presence of a gradient term. Note however, that if one sets $\nu = 0$, then the first equation becomes a Hamilton-Jacobi transport equation for the order parameter S, which is certainly appropriate for a phase field equation, which is intended to model the movement of an interface. Therefore one can consider this equation as a regularized transport equation. Since Hamilton-Jacobi equations are hyperbolic, the new phase field equations have properties in between hyperbolic and parabolic equations. This is why we call these equations hybrid models.

To understand the mathematical properties of these equations, we derived precise asymptotic expansions for $\nu \to 0$ for both equations. Moreover, we intensively tested the first equation numerically, whereas the second equation has not been submitted to a numerical test as yet. We shall present our analytic and numerical results.

These analytic and numerical results have given us strong confidence in these new models. Still, difficult mathematical problems remain. Not the least of these problems is to prove existence of solutions in more than one space dimension.

This is in part joint work with Peicheng Zhu.

H.-D. Alber, *Asymptotics and numerical efficiency of the Allen-Cahn model for phase interfaces with low energy in solids.* Continuum Mech. Thermodyn. **29,3** (2017), 757–803. doi:10.1007/s00161-017-0558-x, arxiv.org/abs/1505.05442

Ort: Mathematik Ost, 40.03.003   Zeit: Freitag, 13.12.19 um 12:00 Uhr

Zu diesem Vortrag sind Sie herzlich eingeladen.

gez. Anja Schlömerkemper