Two-day Workshop on Geometric Analysis, 3–4 July 2025

Schedule:

- Thursday, 3 July, Seminarraum, Emil-Fischer-Str. 41
  - 9:00: Tobias König, Stability with explicit constants for reverse Sobolev inequalities on the sphere
  - 12:00: Azahara DelaTorre, Nondegeneracy of the bubble in a fractional and singular 1D Liouville equation
  - 14:15: Juncheng Wei, A complete solution to Brezis' open question 1.1
  - 16:00: Guofang Wang, A new Yamabe problem
- Friday, 4 July, Seminarraum, Emil-Fischer-Str. 40
  - 9:30: Jonas Peteranderl, TBA
  - 11:00: João Henrique Andrade, Geometry from diffuse interfaces: A PDE approach to isoperimetric clusters

Abstracts:

João Henrique Andrade: Geometry from diffuse interfaces: A PDE approach to isoperimetric clusters

This talk explores the geometric and variational structure behind solutions to the vector-valued Allen–Cahn equation on compact Riemannian manifolds. In the low-temperature regime, solutions concentrate near optimal multi-phase configurations, generalizations of classical isoperimetric sets whose interfaces approximate minimal or constant-mean-curvature hypersurfaces. Despite recent breakthroughs, including the resolution of the multi-bubble conjecture in Euclidean space, the general structure of multi-isoperimetric clusters on manifolds remains elusive: no explicit formula is known for their profile, and even existence requires delicate regularity theory. We present an alternative way to bypass these challenges by combining Γ-convergence methods with a topological construction to encode the manifold's geometry into approximate almost minimizers. This yields lower bounds on the number of solutions in terms of some well-known topological invariants such as the Lusternik–Schnirelmann category and the sum of Betti numbers, revealing deep ties between nonlinear PDEs, topology, and geometric measure theory.

Azahara DelaTorre: Nondegeneracy of the bubble in a fractional and singular 1D Liouville equation

In this talk, we will focus on a stationary fractional nonlinear equation with exponential nonlinearity defined on the whole real line in the presence of a singular term. This particular equation appears as a limit problem in physical models describing galvanic corrosion phenomena in simple electrochemical systems. We will show the non-degeneracy of its solutions for all possible values of the parameter that determines the singularity.

We will use conformal transformations to rewrite the linearized equation as a Steklov eigenvalue problem posed in a Lipschitz bounded domain, which is defined either by the intersection or the union of two disks. We conclude by proving the simplicity of the corresponding eigenvalue.

The talk is based on work done in collaboration with G. Mancini, A. Pistoia, and L. Provenzano.

Tobias König: Stability with explicit constants for reverse Sobolev inequalities on the sphere

For $s - \frac{n}{2} \in (0,1) \cup (1,2)$, a reverse-type Sobolev inequality of order $2s$ holds on the sphere $\mathbb{S}^n$. In my talk, I will discuss recent results by Gong-Yang-Zhang (arXiv:2503.20350) and myself (arXiv:2504.19939) on the quantitative stability of this inequality. Implementing the classical proof strategy by Bianchi and Egnell is non-trivial here because the underlying operator A_{2s} is not positive definite when $s > \frac{n}{2}$. Remarkably, the case $s - \frac{n}{2} \in (1,2)$ constitutes the first example of a Sobolev-type stability inequality (i) whose best constant is explicit and (ii) which does not admit an optimizer.

Jonas Peteranderl: TBA

Guofang Wang: A new Yamabe problem

Starting from a new Sobolev inequality between the total Q curvature and the total scalar curvature we consider a New Yamabe problem: Finding a conformal metric with a constant quotient between the Q curvature and the scalar curvature. The talk is based on a joint project with Yuxin Ge (Toulouse) and Wei Wei (Nanjing and Freiburg)

Juncheng Wei: A complete solution to Brezis' open question 1.1

In 2023, H. Brezis published a list of his "favorite open problems" that he raised through his career and that have resisted so far. We completely solve the first one– open problem 1.1. in Brezis' open problem list: the existence of solutions to the long-standing Brezis-Nirenberg problem on a three-dimensional ball:

$$\Delta u + \lambda u + u^5 = 0 \text{ in } \mathbb{B}_1, \quad u = 0 \text{ on } \partial\mathbb{B}_1.$$

The key idea is to use the sign-changing solutions to the Yamabe problem. (Joint work with L. Sun and W. Yang.)