CORRIGENDUM: SEMIDEFINITE PROGRAMS: NEW SEARCH DIRECTIONS, SMOOTHING-TYPE METHODS, AND NUMERICAL RESULTS

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Abstract. We correct an error in Lemma 4.2 of [KANZOW AND NAGEL, *Semidefinite programs: New search directions, smoothing-type methods, and numerical results*, SIAM J. Optim 13, 2003, pp. 1–23]. With this correction, all results in that paper remain true.

Key words. semidefinite programs, smoothing-type methods, Newton's method, global convergence, superlinear convergence

AMS subject classifications. 90C22, 90C46

Let $L_A(X) := AX + XA$ be the Lyapunov operator associated to a given matrix $A \in \mathbb{R}^{n\times n}$. In Lemma 4.2 (c) of [1] we asserted that the composition $L_A \circ L_B$ is strongly monotone for two symmetric positive definite matrices $A, B \succ 0$. While the given proof holds if A and B commute, inequality (4.8) in [1], namely

$$\mathrm{tr}(X(BA + AB)X) \geq 0$$

does not hold in general for all $X \in \mathcal{S}^{n\times n}$: Setting

$$A := \begin{pmatrix} 10 & 5 \\ 5 & 6 \end{pmatrix}, \quad B := \begin{pmatrix} 1 & 2 \\ 2 & 6 \end{pmatrix}, \quad X := \begin{pmatrix} 15 & -7 \\ -7 & 3 \end{pmatrix},$$

an easy calculation shows that $A, B \succ 0$ but $\mathrm{tr}(X(BA + AB)X) = -588 < 0$. Therefore, to prove part (c), (d) of Lemma 4.2 in [1], we have to add the assumption

$$BA + AB \succeq 0. \tag{0.1}$$

In Proposition 4.4 we used Lemma 4.2 (d) with $A := E - S$ and $B := E - X$ (where $E := \left(X^2 + S^2 + 2\tau^2 I\right)^{1/2}$ for some $\tau > 0$) in order to show that a certain linear mapping is bijective. We now have to prove that these matrices satisfy inequality (0.1). This will be done in the following Lemma.

LEMMA 0.1. *Let* $E := \left(X^2 + S^2 + 2\tau^2 I\right)^{1/2}$, $A := E - S$ *and* $B := E - X$. *Then* $BA + AB \succeq 0$.

Proof. We have

$$\begin{aligned} BA + AB &= (E - X)(E - S) + (E - S)(E - X) \\ &= E^2 - XE - ES + XS + E^2 - SE - EX + SX \\ &= X^2 + S^2 + 2\tau^2 I - E(X + S) - (X + S)E + E^2 + XS + SX \\ &= (X + S - E)^2 + 2\tau^2 I \succeq 0. \end{aligned}$$

This completes the proof. □

REFERENCES

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[1] C. Kanzow and C. Nagel, *Semidefinite programs: New search directions, smoothing-type methods, and numerical results*, SIAM J. Optim.. 13 (2003), pp. 1–23.