Some commentators who write about gold forward rates (GOFOs) seem to lack even the most basic knowledge about the subject. For instance, Fekete Research has recently published and endorsed a paper (reference [1]) containing the claims that “gold’s rate of interest is most often noted as GOFO”, and that a “GOFO quoted higher than LIBOR means that lending gold is more profitable than lending dollars”. This essay will explain in detail why these statements could not be further from the truth. That writers behind an investment newsletter specialized in gold forward observations make such elementary mistakes and do not even correct them once they have been pointed out to them (reference [2]), should be a great cause of concern to researchers and investors who rely on such services.

To get the facts straight: Gold's interest rate is the gold lease rate (GLR). The gold forward rate (GOFO), however, is a way of expressing the forward price of gold in terms of a percentage above or below the current spot price. We will see that GOFO can also be interpreted as a swap rate, but it certainly is not “gold's rate of interest”. To precious metals professionals, these are well-known facts. Nevertheless, more than once I have seen commentary on GOFO and related topics from which it was obvious that the writers did not possess this most elementary knowledge. I will therefore explain in more detail what GOFO and GLR are; but also how GOFO can be defined in different equivalent ways, and why the equation LIBOR = GOFO + GLR holds.

GOFO as a premium of the forward over spot
GOFO is a description of the forward price of gold in terms of a percentage or “premium” above or below the current price. For example, if the current market price (spot price) of gold is 1,200 U.S.-Dollars (USD) per ounce (oz) of gold, and the one year forward price is USD 1,212, then 1-year GOFO is 1% (positive 1%). If the spot price of gold is again USD 1,200, but the one year forward price is USD 1,176, then 1-year GOFO is –2% (negative 2%). In the first case, 1-year forward gold can said to be in contango, and in the second case in backwardation.

The gold lease rate (GLR)
The interest rate at which market participants can borrow gold without posting any collateral is called the gold lease rate (GLR). It is usually denominated relative to the Dollar amount of borrowed gold. For example, if the one year GLR was at 2% and gold spot was at USD 1,200, then someone could now borrow 1 oz of gold for one year, and they would have to return 1 oz of gold plus USD 24 in one year's time. While GLR could also be expressed as a percentage of gold ounces that have to be returned, this is not commonly done. Nonetheless, it is obvious from this definition that GLR is the interest rate for borrowing gold.

LIBOR = GOFO + GLR
While GLR is gold's interest rate, the London Interbank Offered Rate (LIBOR) is the Dollar's. Again, this rate applies when no collateral is posted. In the following, we will assume that a market participant, for instance a large bullion bank in London, could borrow, respectively lend, gold at GLR, or Dollars at LIBOR. We will ignore markets spreads, default risk, margin requirements, and any transaction or storage costs. Under these assumptions, it can now be shown that GOFO and GLR add up to LIBOR.
A no-arbitrage argument
To make this easier, assume now that the equation was not true and that GOFO was at 1%, GLR was at 2%, and LIBOR at 4% (below it will become clear that the actual rates do not matter). Further, assume that a market participant with no initial capital borrows 1 oz of gold for one year and immediately sells it at the spot price of currently USD 1,200. She can now lend her cash out at LIBOR for one year. Furthermore, she can enter a gold forward contract for one year (at no immediate cost) with a forward price of USD 1,212, since GOFO is at 1%. The resulting cash flow at the start of this strategy is obviously zero. We will now show that this strategy, that initially costs nothing, will return a positive risk-free profit.

One year on
After one year's time, our market participant receives USD 1,248 from her LIBOR investment. Using the forward contract, she pays USD 1,212 to buy 1 oz of gold. She then returns the gold ounce to the gold lender from which she borrowed it originally and to whom she now has to pay the borrowing fee of USD 24, as GLR stood at 2%. The resulting cash flow is therefore:

\[
\text{USD } 1,248 - \text{USD } 1,212 + 1 \text{ oz gold} - 1 \text{ oz gold} - \text{USD } 24 = \text{USD } 12.
\]

Expressed differently, after one year, our market participant would have made USD 12 starting with zero initial capital. Since nothing would prevent her from running the same strategy for a thousand ounces, or more, hence making USD 12,000, or more, out of nothing, it is obvious that this strategy is too good to be true. It thus is an arbitrage and should not exist in a liquid and efficient market (see also my recent articles “Fekete's Arbitrage Fallacy” (reference [3]) and “The Time Value Of Gold – Ignore It At Your Own Peril” (reference [4]) for more comprehensive explanations of the concept of “arbitrage”). However, from the bold parts of the equation, it is clear that this risk-free profit out of nothing only came into existence because GOFO, at 1% or USD 12, and GLR, at 2% or USD 24, added up to less than LIBOR, at 4% or USD 48. Therefore, under the assumption of no arbitrage opportunities in the market, this cannot not be the case.

What if it adds up to more?
If GOFO and GLR had added up to more than LIBOR, the earlier profit of USD 12 would have turned into a loss. However, a similar no-arbitrage argument would still work. This time, our market participant would have to pursue the strategy where she would borrow at LIBOR to buy 1 oz of gold at spot, only to immediately lend it out at GLR, while going short a forward contract at no cost. In the earlier equation of the resulting cash flow after one year, this would turn plus into minus and minus into plus, such that the profit of this strategy would again be positive, thus completing the proof. We hence showed for the one year maturity, that LIBOR = GOFO + GLR. This equation is the reason why the London Bullion Market Association (LBMA) publishes the value “LIBOR MINUS GOFO” as “derived gold lease rate” (reference [6]). For instance, on December 12, 2013, those rates were for the one year maturity given as LIBOR at 0.57630%, GOFO at 0.13667%, and therefore the derived GLR at 0.43963%.

Backwardation: GOFO smaller than zero, GLR greater than LIBOR
From LIBOR = GOFO + GLR, it follows immediately that backwardation – that is the state were the forward price is smaller than the spot price and hence GOFO smaller than zero – means that gold's interest rate GLR is higher than the Dollar interest rate LIBOR. The equation also is one of the main reasons why selling gold at spot, and buying it back at a lower forward price, constitutes no arbitrage. A detailed explanation of this can be found in my article “Faux Gold Arbitrage” (reference [3]).
GOFO higher than LIBOR
The equation that we have just derived also shows that if GOFO is higher than LIBOR, then GLR has to be negative. The LBMA has reported negative gold lease rates for shorter maturities in the past, for instance at the start of 2013. Whether in theory or in practice, negative gold lease rates mean that a market participant would have to pay(!) money to be allowed to lend gold to someone. The earlier quoted statement from the Fekete Research website saying that “GOFO quoted higher than LIBOR means that lending gold is more profitable than lending dollars” therefore is incorrect when LIBOR is positive, which it generally is.

GOFO as a swap rate
On the internet pages (reference [6]) of the LBMA, the following definition of GOFO can be found:

GOFO stands for Gold Forward Offered Rate. These are rates at which contributors are prepared to lend gold on a swap against US dollars. Quotes are made for 1-, 2-, 3-, 6- and 12-month periods.

It is possibly not entirely clear from this definition that GOFO is paid by the gold lender, who is the Dollar borrower, on the borrowed amount of Dollars, which at the time of the agreement is identical to the spot price of the amount of gold leased. For instance, if the spot price of gold stands at USD 1,200 and GOFO at 1%, and someone wants to borrow U.S.-Dollars against one ounce of gold, they will receive USD 1,200 at the start of the year, while handing over their 1 oz of gold to the swap's counterparty. At the end of the year, they will pay USD 1,212 to the counterparty to receive back their 1 oz of gold. If GOFO was to be interpreted as an interest rate, then it follows from this definition as a swap rate that it is an interest rate on a Dollar loan if gold is posted as a collateral.

Equivalence of the two definitions
The definition of GOFO as a premium above/below spot describing the forward price, and the definition as a rate in a Dollars-for-gold swap, are equivalent. To prove this, we can simply consider a situation where this was not the case. If we can then show that in this situation a market participant could make “money out of nothing” – an arbitrage – then we are done if we assume that arbitrage opportunities in efficient markets immediately disappear. Therefore, assume now that the spot price of gold was USD 1,200 and that the 1-year forward price was USD 1,224, so GOFO defined as a premium was at 2%. However, assume that the earlier described swap rate was at 1% instead. Starting with zero capital, a market participant can now borrow USD 1,200 at LIBOR of, say, 4%, and buy 1 oz of gold for the spot price of USD 1,200. She can now enter the swap agreement, where she lends 1 oz of gold and borrows USD 1,200, and she can invest the received cash at the LIBOR rate for one year. Furthermore, she can also go short a 1 oz gold forward contract with maturity one year at the forward price of USD 1,224. Since, at the time of the agreement, a forward costs nothing, the resulting cash flow at the start of this strategy would be zero.

One year on
Our investor would now receive USD 1,248 from the swap money of USD 1,200 that she had received and immediately invested at LIBOR of 4% a year. Regarding the swap agreement, she would now have to pay USD 1,212, as the swap rate was 1%. She would receive 1 oz of gold from the swap. She would have to pay 1 oz of gold because of her short forward contract, and she would receive USD 1,224 for this in return, as this was the agreed forward price (GOFO at 2%). She would now have to pay USD 1,248 to close the original loan (that she took out to buy the gold) plus interest at LIBOR of 4%. The resulting cash flow would therefore be:
USD 1,248 – USD 1,212 + 1 oz gold – 1 oz gold + USD 1,224 – USD 1,248 = USD 12.

This would constitute an arbitrage. From the bold parts of the equation, it is clear that this “free lunch” only existed, because GOFO, at 2%, was higher than the swap rate at 1% (the LIBOR rate cancelled out). Therefore, under the assumption of no arbitrage opportunities, this should not be the case. As earlier, when we proved LIBOR = GOFO + GLR, the strategy could again be inverted to show that, under the assumption of no-arbitrage, GOFO could also not be lower than the swap rate, and therefore equality of the two rates was established.

A final remark
In the sea of information and misinformation that we call the “internet”, it can be very difficult to tell good from bad, or competent from incompetent. Unfortunately, the “gold investment community” seems to have no lack of pseudo experts that provide sloppy research or outright factual misinformation to unsuspecting investors. Nonetheless, there are many good commentators that provide original and high quality commentary. To find them, fact checking, as well as checking of qualifications or references, is essential, as tedious as it can be. However, everyone can err – whether it is a maths professor or a professional bullion trader. In the end, we have to use our own logic to judge.

References


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