Julius-Maximilians-UNIVERSITÄT WÜRZBURG

Lehrstuhl für MATHEMATIK IX

Übungen zur Optimale Steuerung (A. Borzì), Blatt 1

SoSe 2019 1 St., Mi 13-14, SE 30

(Calculus of Variation ODE, Optimal Control ODE, Optimal Control PDE)

(1) Use the method of Lagrange multipliers to solve the following constrained optimization problem

$$\min f := x^2 + y^2 + z^2$$
$$\text{s.t. } \phi := xy + 1 - z = 0$$

(2) Of all rectangular parallelepipeds which have sides parallel to the coordinate planes and which are inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \qquad a, b, c > 0,$$

determine the dimensions of that one which has the largest volume.

(3) Of all parabolas which pass through the points $(0, 0)$ and $(1, 1)$, determine that one which, when rotated about the x-axis, generates a solid of revolution with least possible volume between $x = 0$ and $x = 1$. (Notice that the equation may be taken in the form $y = x + cx(1 - x)$, when c is to be determined.)

(4) Give the proof of the Extreme Value Theorem: Let X be a compact metric space and $f : X \to \mathbb{R}$ continuous. Then there are $x_0, x_1 \in X$ such that

$$f(x_0) = \min_{x \in X} f(x) \qquad f(x_1) = \max_{x \in X} f(x).$$

(5) Suppose that I is an interval and $f : I \to \mathbb{R}$ is twice differentiable. Prove that

1. f is convex if and only if $f''(x) \geq 0$ for all $x \in I$.
2. If $f''(x) > 0$ for all $x \in I$, then f is strictly convex.
3. f is convex if and only if the graph of f is above all of its tangent lines, that is, if $f(y) \geq f(x) + f'(x)(y - x)$ for all $x, y \in I$.