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UNIVERSITÄT
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Lehrstuhl für
MATHEMATIK IX

Übungen zur Optimale Steuerung (A. Borzì), Blatt 3

SoSe 2019 1 St., Mi 13-14, SE 30

(Calculus of Variation ODE, Optimal Control ODE, Optimal Control PDE)

(1) A chain is suspended from two thin hooks that are 16 meters apart on a horizontal line. The chain itself consists of 20 links of stiff steel. Each link is one meter in length. Determine the equilibrium shape of the chain.

(Hint: the link i spans an x distance of x_i and a y distance of y_i such that $x_i^2 + y_i^2 = 1$, consider the minimization of the gravitational potential energy.)

(This and the next example are taken from the book: Luenberger, David G., Ye, Yinyu, Linear and Nonlinear Programming, Springer, 2016.)

(2) Consider a discrete probability density corresponding to a measured value taking one of n values $x_1, x_2, \dots, x_n$. The probability associated to x_i is p_i. The p_i's satisfy the conditions $p_i \geq 0$ and $\sum_{i=1}^n p_i = 1$. The entropy of such a density is given by

$$H = -\sum_{i=1}^{n} p_i \, \log(p_i).$$

With the probability density, we can compute the mean value: $m = \sum_{i=1}^n p_i \, x_i$.

If the value of mean is known to be m, find the density p_i, $i = 1, \dots, n$, that maximizes H.

(3) Discuss the extremal points of this problem

$$\min f(x_1, x_2) := x_1^3 - x_1^2 \, x_2 + 2 \, x_2^2$$
$$s.t. \qquad x_1 \geq 0, \qquad x_2 \geq 0.$$

(4) Consider a linear system of two equations $A\,x = 0$ given by

$$a\,x_1 + b\,x_2 + x_3 = 0$$
$$c\,x_1 + d\,x_2 + x_3 = 0$$

where $ad - bc \neq 0$. Consider (x_1, x_2, x_3) in two sets of variables $y = (x_1, x_2)$ and $u = x_3$. Determine the mapping $u \mapsto y$ and its first derivative.

Discuss the minimization of the function $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$, subject to the equality constraint $Ax = 0$, in terms of the variable u.

(5) Investigate the optimization problem

$$\begin{aligned} \min f(x_1, x_2) &:= x_1 + x_2 + x_3 \\ s.t. \qquad c = g(x_1, x_2, x_3) &:= x_1^2 + x_2^2 + x_3^2 - 1, \end{aligned}$$

where $c \in \mathbb{R}$ represents a perturbation of the equality constraint $g(x) = 0$. Find the global minimizer of this problem, and the corresponding Lagrange multiplier. Discuss the sensitivity of the minimizing solution with $c = 0$.