Julius-Maximilians-
UNIVERSITÄT
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Lehrstuhl für
MATHEMATIK IX

Übungen zur Optimale Steuerung (A. Borzì), Blatt 4

SoSe 2019 1 St., Mi 13-14, SE 30

(Calculus of Variation ODE, Optimal Control ODE, Optimal Control PDE)

(1) Prove the following:

Let $Q \in \mathbb{R}^{n\times n}$ and $A \in \mathbb{R}^{m\times n}$ be given, and it holds that $rank(A) = m$, and Q is positive definite in the subspace $ker(A)$. Then the following matrix is invertible:

$$\begin{pmatrix} Q & A^T \\ A & 0 \end{pmatrix}.$$

(2) Let X be a normed space and $f : X \to \mathbb{R}$ be a function. We say that f is lower semi-continuous at x_0 if and only if for every $\epsilon > 0$ there exists $\delta > 0$ such that

$$f(x) - f(x_0) > -\epsilon,$$

whenever $\|x - x_0\| < \delta$.

Prove that f is lower semi-continuous at x_0 if and only if for every sequence x_n with $\lim_{n\to\infty} x_n = x_0$, it follows that $\liminf_{n\to\infty} f(x_n) \geq f(x_0)$.

(3) Let (f_j), (g_j) be any two strongly convergent sequences in an arbitrary infinite dimensional Hilbert space H with scalar product $(\cdot,\cdot)_H$, and (h_j) ,(k_j) be any two weakly convergent sequences in H. Prove or find a counterexample:

a) The sequence $(f_j, g_j)_H$ is always convergent.

b) The sequence $(f_j, h_j)_H$ is always convergent.

c) The sequence $(h_j, k_j)_H$ is always convergent.

(4) Find a function in $C[0,1]$ that is not contained in $H^1(0,1)$. Find a sequence (v_n) in $C_0^\infty(0,1)$ which converges to the constant function $y = 1$ in the $L^2(0,1)$ sense.

(5) Consider the functional

$$J(y) = \int_0^1 (1+x)\,(y'(x))^2\,dx,$$

where $y \in C^2[0,1]$, and $y(0) = 0$ and $y(1) = 1$. Of all functions of the form

$$y(x) = x + c_1\,x\,(1-x) + c_2\,x^2\,(1-x),$$

where c_1 and c_2 are constants, find the one that minimizes J.