Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Lehrstuhl für
MATHEMATIK IX

Übungen zur Optimale Steuerung (A. Borzì), Blatt 5

SoSe 2019 1 St., Mi 13-14, SE 30

(Calculus of Variation ODE, Optimal Control ODE, Optimal Control PDE)

(1) Prove the following lemma: If $h \in C^1[0,1]$ and $h(0) = 0$, then

$$\|h\|_\infty \leq 2\Big[\int_0^1 (h'(x))^2\Big]^{\frac{1}{2}}.$$

(2) Prove the following lemma: If $f : \mathbb{R} \to [0, +\infty]$ is a convex function, then the functional $J : L^1(a, b) \to [0, +\infty]$ given by

$$J(u) = \int_a^b f(u(x))\, dx$$

is weakly lower semi-continuous in $L^1(a, b)$.

(3) Let X and Y be Banach spaces, and $F : X \to Y$ be a function, defined on an open set $D \subset X$. Show that

1. If $F : X \to Y$ is differentiable at $x \in X$, then it is also continuous.
2. The notion of differentiability and the derivative of F do not change if the norms $\|\cdot\|_X$ and $\|\cdot\|_Y$ are replaced by equivalent norms $\|\cdot\|'_X$ and $\|\cdot\|'_Y$.

(4) Consider the function $f : L^2(0, \pi) \to L^2(0, \pi)$ given by $f(u) = \sin(u)$. Compute its Gâteaux derivative. Show that this map is not Fréchet differentiable in $L^2(0, \pi)$. (Consider $u = 0$.)

(5) Discuss the functional

$$J(y) = \int_0^1 \left(x^2\,(y(x))^2 + (y'(x))^4\right)\, dx,$$

defined in $C^1[0, 1]$, and compute its first and second variation.