Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Lehrstuhl für
MATHEMATIK IX

Übungen zur Optimale Steuerung (A. Borzì), Blatt 6

SoSe 2019 1 St., Mi 13-14, SE 30

(Calculus of Variation ODE, Optimal Control ODE, Optimal Control PDE)

(1) Consider the functional

$$J(y) = \int_0^\pi (y(x))^2 \, (1 - (y'(x))^2) \, dx,$$

in the space $C^1[0, \pi]$, and the boundary conditions $y(0) = 0$ and $y(\pi) = 0$. Show that the function $y(x) = 0$ is a weak local minimizer of J in the given space. Show that $y(x) = 0$ is not a strong local minimizer.

(2) Find all extremals of the functional given by

$$J(y) = \int_0^{2\pi} ((y'(x))^2 - (y(x))^2) \, dx,$$

in the space $C^1[0, 2\pi]$, and the boundary conditions $y(0) = 1$ and $y(2\pi) = 1$.

(3) Find all extremals of the functional given by

$$J(y) = \int_1^2 y'(x) \, (1 + x^2 \, y'(x)) \, dx,$$

in the space $C^1[1, 2]$, and the boundary conditions $y(1) = 3$ and $y(2) = 5$.

(4) Discuss the variational problem with the functional

$$J(y) = \int_a^b \left((y(x))^2 + 2 \, x \, y(x) \, y'(x) \right) \, dx,$$

in the space $C^1[a, b]$, and the boundary conditions $y(a) = A$ and $y(b) = B$.

(5) The drag force on an x-axial symmetric object moving along this axis with velocity v in a gas medium of density ρ is given by

$$F(y) = 4\pi\rho v^2 \int_0^\ell y(x) \, (y'(x))^3 \, dx,$$

where $[0, \ell]$ is the size of this object in the x direction, and its circular cross section at $x = \ell$ has a given radius R. Find the profile y along x of the object that minimizes the drag.