Übungen zur Optimale Steuerung (A. Borzì), Blatt 7

SoSe 2019 1 St., Mi 13-14, SE 30

(Calculus of Variation ODE, Optimal Control ODE, Optimal Control PDE)

(1) Find the minimizer of the functional

$$J(y) = \int_0^\pi (y'(x))^2 \, dx,$$

in the space $C^1[0,\pi]$, and subject to the condition

$$\int_0^\pi (y(x))^2 \, dx = 1,$$

and the boundary conditions $y(0) = 0$ and $y(\pi) = 0$.

(2) Find the broken extremals (if any) of the functional given by

$$J(y) = \int_0^4 (y'(x) - 1)^2 \, (y'(x) + 1)^2 \, dx,$$

with the boundary conditions $y(0) = 0$ and $y(4) = 2$.

(3) Derive the Hamiltonian equations for the functional

$$J(y_1, y_2) = \int_0^\pi \left(2y_1(x)y_2(x) - 2y_1^2(x) + (y_1'(x))^2 - (y_2'(x))^2\right) \, dx.$$

(4) Show that it is not possible to construct the Hamiltonian operator for the functional

$$J(y_1, y_2) = \int_a^b y_1^2(x) \, y_2^2(x) \, \left(x^2 + y_1'(x) + y_2'(x)\right) \, dx.$$

(5) Discuss E. Noether's theorem and present an example.