(1) Find the minimizer of the functional

\[ J(y) = \int_0^\pi (y'(x))^2 \, dx, \]

in the space \( C^1[0, \pi] \), and subject to the condition

\[ \int_0^\pi (y(x))^2 \, dx = 1, \]

and the boundary conditions \( y(0) = 0 \) and \( y(\pi) = 0 \).

(2) Find the broken extremals (if any) of the functional given by

\[ J(y) = \int_0^4 (y'(x) - 1)^2 (y'(x) + 1)^2 \, dx, \]

with the boundary conditions \( y(0) = 0 \) and \( y(4) = 2 \).

(3) Derive the Hamiltonian equations for the functional

\[ J(y_1, y_2) = \int_0^\pi \left( 2y_1(x)y_2(x) - 2y_1^2(x) + (y_1'(x))^2 - (y_2'(x))^2 \right) \, dx. \]

(4) Show that it is not possible to construct the Hamiltonian operator for the functional

\[ J(y_1, y_2) = \int_a^b y_1^2(x) \, y_2^2(x) \left( x^2 + y_1'(x) + y_2'(x) \right) \, dx. \]

(5) Discuss E. Noether’s theorem and present an example.