Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Lehrstuhl für
MATHEMATIK IX

Übungen zur Optimale Steuerung (A. Borzì), Blatt 8

SoSe 2019 1 St., Mi 13-14, SE 30

(Calculus of Variation ODE, Optimal Control ODE, Optimal Control PDE)

(1) Show that this set of admissible controls is closed, convex and bounded in $L^2(a,b)$

$$U_{ad} = \left\{ u \in L^2(a,b) \,|\, \underline{u} \le u(x) \le \overline{u} \ \text{ a.e. in } (a,b) \right\},$$

where $-\infty < \underline{u} < \overline{u} < +\infty$.

(2) Consider the following optimal control problem

$$\begin{aligned} &\min J(y,u) := \frac{1}{2}\int_0^1 \left(|y(x)|^2 + |u(x)|^2 \right) dx \\ &\text{s.t.} \quad y'(x) = u(x), \qquad y(0) = 0, \\ &u \in L^2(a,b). \end{aligned}$$

Define the reduced cost functional and prove that it is strictly convex. Show that this fact implies uniqueness of the optimal control.

(3) Find all extremals of the following optimal control problem

$$\begin{aligned} &\min J(y,u) := \int_0^1 y(x)\, u(x)\, dx \\ &\text{s.t.} \quad y'(x) = u(x), \qquad y(0) = 0, \\ &u \in U_{ad} := \{ u \in L^2(0,1) \,|\, u(x) \in [-1,1] \ \text{ a.e. in } (0,1) \}. \end{aligned}$$

(4) Discuss existence, uniqueness, and regularity of solutions to the following optimal control problem

$$\begin{aligned} &\min J(y,u) := \frac{1}{2}\int_a^b |y(x) - y_d(x)|^2\, dx + \frac{\nu}{2}\int_a^b |u(x)|^2\, dx + (y(b))^2 \\ &\text{s.t.} \quad y'(x) = f(x)\, y(x) + u(x), \qquad y(a) = y_a, \\ &u \in L^2(a,b), \end{aligned}$$

where $\nu > 0$ and $f \in C^1[a,b]$, and $y_d \in L^2(a,b)$ is a given target trajectory.

(5) Discuss existence, uniqueness, and regularity of solutions to the following optimal control problem

$$\begin{aligned}
&\min J(y,u) := \frac{1}{2}\int_a^b |y(x) - y_d(x)|^2\,dx + \frac{\nu}{2}\int_a^b |u(x)|^2\,dx \\
&\text{s.t.} \quad -y''(x) = u(x), \qquad y(a) = y_a,\ y(b) = y_b, \\
&u \in L^2(a,b),
\end{aligned}$$

where $\nu > 0$ and $y_d \in L^2(a,b)$ is a given target configuration.