A multigrid optimization algorithm for the numerical solution of quasilinear variational inequalities involving the p-Laplacian

In this work we propose a multigrid optimization algorithm (MG/OPT) for the numerical solution of a class of quasilinear variational inequalities of the second kind. This approach is enabled by the fact that the solution of the variational inequality is given by the minimizer of a nonsmooth energy functional, involving the p-Laplace operator. We propose a Huber regularization of the functional and a finite element discretization for the problem. Further, we analyze the regularity of the discretized energy functional, and we are able to prove that its Jacobian is slantly differentiable. This regularity property is useful to analyze the convergence of the MG/OPT algorithm. In fact, we demonstrate that the algorithm is globally convergent by using a mean value theorem for semismooth functions. Finally, we apply the MG/OPT algorithm to the numerical simulation of the viscoplastic flow of Bingham, Casson and Herschel–Bulkley fluids in a pipe. Several experiments are carried out to show the efficiency of the proposed algorithm when solving this kind of fluid mechanics problems.