Optical Flow by Optimal Control

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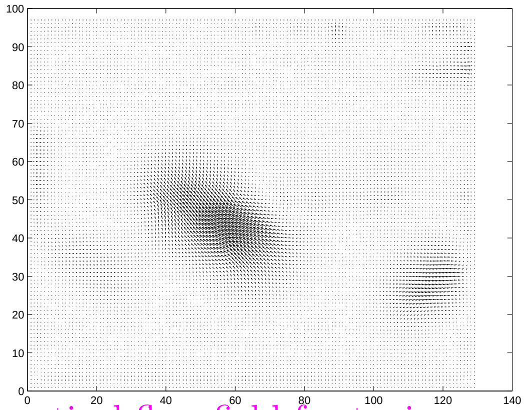
Objective: A problem of central significance in computer vision is to capture the motion of objects of interest in a sequence of images. The optical flow field of apparent velocities is determined here by an optimal control approach.

Applications: Information about the spatial arrangement of objects and the rate of change of this arrangement can be computed, e.g., for medical imaging, vision robotics, terrain mapping, and particle image velocimetry. A benchmark problem is to compute the optical flow field for a taxi sequence:

first frame of taxi sequence



optical flow field for taxi scene

Required: velocity field $\boldsymbol{w}$ along which constant brightness I is convected, i.e., $\nabla I \cdot \boldsymbol{w} + I_t = 0$. The ambiguity implies a selection process is needed to specify $\boldsymbol{w}$.

Variational Principle: resolve ambiguity imposing minimal divergence, $|\nabla \boldsymbol{w}| = \min$, and thereby avoid unnatural light sources.

Optimal Control Formulation: given images $Y_k : \boldsymbol{R}^N \supset \Omega \to \boldsymbol{R}$, determine the optical flow field $\boldsymbol{w} : \Omega \times [0,T] \to \boldsymbol{R}^N$ and (without data differentiation) a regularized intensity field $I : \Omega \times [0,T] \to \boldsymbol{R}$ by minimizing:

$$J(\boldsymbol{w}, I) = \int_0^T \int_\Omega \left[\sum_k \delta(t - t_k)|I - Y_k|^2 + \varphi(|\boldsymbol{w}_t|) + \psi(|\nabla \boldsymbol{w}|) + \gamma |\nabla \cdot \boldsymbol{w}|^2 \right] d\boldsymbol{x}dt$$

$$\text{subject to } \nabla I \cdot \boldsymbol{w} + I_t = 0, \quad \Omega \times [0,T].$$

Optimality System: Use second order TVD schemes to solve hyperbolic equations for I and a Lagrange multiplier p for a given $\boldsymbol{w}$, and use a second order multigrid scheme to solve an elliptic system for $\boldsymbol{w}$ for given I and p. Both a regularized intensity field I and an optical flow field $\boldsymbol{w}$ are achieved at convergence.