Optimal Control of Explosive Systems

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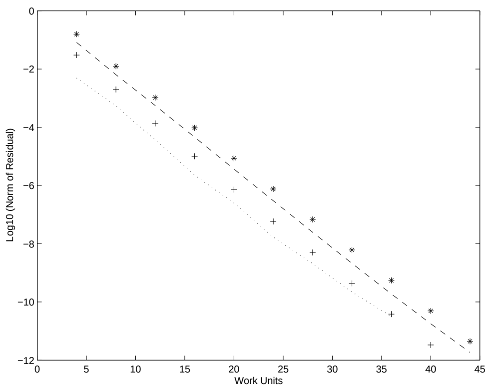
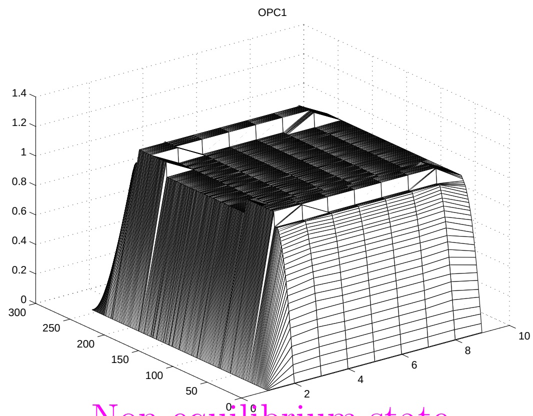
Objective: A challenging mathematical and technological problem is to control explosive systems in order to drive and maintain them in a required non-equilibrium state. This task is achieved here by an optimal control approach.

Applications: Manufacturing of solid fuel systems and optimal combustion processes.



Convergence of the GSN-multigrid. $\nu \in [10^{-9}, 10^{-3}]$.



Non-equilibrium state

Required: heating source term, $f(x,t)$, such that a prescribed non-equilibrium state, $z(x,t)$, is (approx.) reached.

Optimal Control Formulation:

$$\begin{aligned} \min_{f \in L^2(Q)} J(u(f), f), &\quad Q = \Omega \times (0,T) \\ -\partial_t u + \Delta u + \delta e^u = f &\quad \text{in } Q, \\ u = 0 &\quad \text{on } \partial\Omega \times (0,T), \end{aligned}$$

cost functional,

$$J(u,f) = \frac{1}{2} ||u - z||^2_{L^2(Q)} + \frac{\beta}{2} ||e^u - e^z||^2_{L^2(Q)} + \frac{\nu}{2} ||f||^2_{L^2(Q)}.$$

Optimality System: Use a second order nonlinear multigrid method to solve the singular optimal control problem. Both a heating function f and the corresponding non-equilibrium state u are achieved at convergence.