Liebau Phenomenon
Alfio Borzi and Georg Propst

Objective: Accurate numerical investigation of the valveless pumping effect known as Liebau phenomenon.

A valveless pump.

Applications: Pumping in cardiovascular systems, design of micro-pumps.

Model for flow in elastic tubes:

\[
\frac{\partial w}{\partial t} + w \frac{\partial w}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = - \frac{1}{\rho} \frac{\partial p_v}{\partial x}, \quad \frac{\partial p}{\partial t} + w \frac{\partial p}{\partial x} + \rho \alpha^2 \frac{\partial w}{\partial x} = \frac{\partial p_a}{\partial t} + w \frac{\partial p_a}{\partial x}. \tag{1}
\]

Forcing term

\[ p_a(x, t) = p_0 + 2 p_0 \frac{(x - x_L)(x_R - x)}{(x_R - x_L)^2} \sin \omega t. \]

Similar behavior is observed with rigid pipes:

\[
\frac{d}{dt} w_i = p_1^b - p_2^b + p_1^c - p_2^c + p_1^v + p_2^v, \quad \frac{d}{dt} h_i = \frac{A_0}{A_B} w_i, \quad i = 1, 2 \quad w^+(t) = \frac{A_c S_{p \omega}}{A_0} \cos(\omega t + \delta)
\]