Exercises of Numerical PDEs
Sheet 1
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Exercise 1. Let u be a continuous function in $\bar{\Omega}$.

1. Explain what is the support of u and make an example of a set Ω and a continuos function u on $\bar{\Omega}$, such that *supp* u is a bounded set and $supp(u) \subseteq \Omega$.
2. Define $C_0^\infty(\Omega)$.
3. Explain the importance of the weak derivative.
4. Consider the function $u(x) = |x|$. Show that u is continuous but not differentiable on $\mathbb{R}$.
5. Find the weak first derivative of u on $\mathbb{R}$.
6. How is the weak second derivative of u on $\mathbb{R}$?

(Points: 0.5+0.5+0.5+0.5+1+1).

Exercise 2. Show that the Cauchy-Schwarz inequality implies that:

1. $\left(\sum_{i=1}^n |X_i|\right)^2 \leq n \sum_{i=1}^n |X_i|^2$, $X = (X_1, X_2, ..., X_n) \in \mathbb{R}^n$,
2. $\|\cdot\|^2_{L^1(\Omega)} \leq \mu(\Omega)\|\cdot\|^2_{L^2(\Omega)}$. ($\mu$ is the measure)

(Points: 2+2).

Exercise 3. Let $\Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq x+y \leq 1\}$. Show that there exists a positive constant c_* (indipendent of u), such that $u \in H_0^1(\Omega)$

$$\|u\|^2_{L^2(\Omega)} \leq c_* \sum_{i=1}^{2} \left\| \frac{\partial u}{\partial x_i} \right\|^2_{L^2(\Omega)} .$$

Hence deduce that $|u|_{H_0^1(\Omega)} := \{\sum_{i=1}^2 \|\frac{\partial u}{\partial x_i}\|^2_{L^2(\Omega)}\}^{1/2}$ defines a norm on $H_0^1(\Omega)$.

(Points: 6)