Online seminar on discrete Inverse Problems

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Organization: The seminar will be held completely online via Zoom. There will be a weekly meeting between October 18th 2021 and February 10th 2022 besides the two weeks around Christmas and New Year. The Day for the meetings is still TBA, but the seminar will be at 8.00 a.m. Berlin time (UTC + 2).

Participants have to register for a online program (*Modulstudium*) at the University of Würzburg. Due to the online nature of the course and local specialties this will be possible free of charge.

Assessment: The idea of the seminar is that each participant gives a talk (ca. 45 minutes) on a pre-chosen topic. The assessment is based on your talk and the overall participation in the other talks. Given a successful assessment you will receive a certificate at the end.

Contents: This seminar will be an introduction to Inverse Problems in a finite-dimensional setting, i.e. on the solution of ill-conditioned matrix equations

$$Ax = b,$$

mostly based on [3]. We will discuss the need of regularization for such equations and different methods to obtain stable reconstructions. A focus is on sprectral regularization methods, which are of the form

$$C_h = q_h \left(A^* A \right) A^*$$

with a family of functions $(q_h)_{h>0}$ depending on a parameter $h > 0$. We will discuss well-definedness of such estimators, derive error bounds on $\|C_h b - x\|$, and discuss different methods for choosing the parameter $h > 0$. If time permits, we will also touch real-world applications e.g. in ordinary differential equations.

Prerequisites: The seminar requires basic knowledge from analysis and linear algebra, as well as some facts from an introductory lecture on numerical analysis (such as algorithms for the solution of linear systems, least squares and the normal equation, eigenvalues of matrices).

References

[1] H. W. Engl, M. Hanke, and A. Neubauer. *Regularization of inverse problems*, volume 375 of *Mathematics and its Applications*. Kluwer Academic Publishers Group, Dordrecht, 1996.

[2] M. Hanke. *A taste of inverse problems*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2017. Basic theory and examples.

[3] A. Neumaier. Solving ill-conditioned and singular linear systems: a tutorial on regularization. *SIAM Rev.*, 40(3):636–666, 1998.