Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

MUΦ X
Lehrstuhl

Workshop

Differential Geometry and Applications

1. - 2. October 2018

Institute of Mathematics, Chair X
Julius-Maximilians-Universität Würzburg
Campus Hubland Nord
Emil-Fischer-Straße 31, Room 00.018
Germany

Program

Monday 1. October 2018

9:30	Arrival with Coffee & Tea (room 00.018)
10:00 - 11:00	**Fatima Silva Leite**, University Coimbra, Portugal
	Left-invariant sub-Riemannian systems on Lie groups and extremal curves on Stiefel manifolds
11:30	Coffee & Lunch break (Mensateria)
13:30 - 14:30	**Ioan Marcut**, Radboud Universiteit Nijmegen, The Netherlands
	A local model around Poisson submanifolds
15:00 -16:00	**Velimir Jurdjevic**, University of Toronto, Canada
	Optimal control on Lie groups and integrable Hamiltonian systems
19:00	Dinner at Mainmühle

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Tuesday 2. October 2018

9:30	Coffee & Tea (room 00.018)
10:00 - 11:00	**Madeleine Jotz Lean**, Georg-August-Universität Göttingen, Germany
	On ideals in Lie algebroids
11:30	Coffee & Lunch break (Mensateria)
13:30 - 14:30	**Max Wardetzky**, Georg-August-Universität Göttingen, Germany
	Discrete Curvature Functionals — and some of their challenges
15:00 -16:00	**Martin Bordemann**, University de Haute Alsace, Mulhouse, France
	(Bi)modules in smooth deformation quantization: old and new, and (non)formality
19:00	Dinner TBA

Abstracts

Martin Bordemann, University de Haute Alsace, Mulhouse, France

Title: (Bi)modules in smooth deformation quantization: old and new, and (non)formality

Abstract: We shall deal with the simultaneous deformation of associative algebras and their (bi)modules applied to deformation quantization on smooth manifolds. There is an obvious formulation in terms of differential graded Lie algebras already mentioned by D.Arnal et al in 1983. We discuss two particular cases: 1. the space of smooth functions on a submanifold (which has to be coisotropic due to Gabber's Theorem), 2. the space of of smooth fonctions on the total space of a fibered manifold over a Poisson manifold. In both cases we compute the corresponding cohomologies (in terms of differential Hochschild cohomology), related this to the differential geometry (e.g. adapted poly-vectorfields), and check L-infinity formality. This is joint work with Benedikt Hurle, Mulhouse.

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Madeleine Jotz Lean, Georg-August-Universität Göttingen, Germany

Title: On ideals in Lie algebroids

Abstract: I will discuss several notions of ideals and explain which of those notions I consider to be the most useful one. I will discuss quotients by those ideals, their equivalence to multiplicative foliations on Lie groupoids, and how they define sub-representations of Lie algebroids adjoint representations (up to homotopy). Then I will sketch first obstructions to the existence of an infinitesimal ideal system structure on a Lie pair.

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Velimir Jurdjevic, University of Toronto, Canada

Title: Optimal control on Lie groups and integrable Hamiltonian systems

Abstract: This lecture will be about two natural left invariant variational problems on semi-simple Lie groups G that admit an involutive automorphism σ. In such situations, the set of fixed points of σ is a closed subgroup K of G, and the Lie algebra $\mathfrak{g}$ of G admits a Cartan decomposition $\mathfrak{g} = \mathfrak{p} \oplus \mathfrak{k}$ with $\mathfrak{k}$ equal to the Lie algebra of K, and $\mathfrak{p}$ a vector space subject to Lie algebraic conditions

$$[\mathfrak{p}, \mathfrak{p}] \subset \mathfrak{k}, \quad [\mathfrak{p}, \mathfrak{k}] = \mathfrak{p}. \tag{1}$$

The above decomposition defines two natural distributions on G: the first distribution $\mathcal{H}$ is defined as

$$\mathcal{H}(g) = \{gX : X \in \mathfrak{p}\}, \quad g \in G, \tag{2}$$

while the second distribution $\mathcal{A}$ is affine, and is defined by an element $A \in \mathfrak{p}$ with

$$\mathcal{A}(g) = \{g(A + X) : X \in \mathfrak{k}\}, \quad g \in G. \tag{3}$$

In this notation, gX stands for the left translate by g of an element $X \in \mathfrak{g}$. In the first case, any two points in G can be connected by a horizontal curve in $\mathcal{H}$ whenever $\mathfrak{p} + [\mathfrak{p}, \mathfrak{p}] = \mathfrak{g}$, while the same is true in the second case whenever A is regular. In this parlance, curve $g(t)$ in G is an integral curve of a distribution $\mathcal{D}$, or a horizontal curve, if $\frac{dg}{dt} \in \mathcal{D}(g)$. Then a suitable scalar multiple of the Killing form can be used to define the length on the space of horizontal curves, which, in turn, induces a natural sub-Riemannian metric on G that is central to the geometry of the underlying symmetric space G/K . In the second case, a negative multiple $\langle\ ,\ \rangle$ of the Killing form $\mathrm{Tr}(\mathrm{ad}\, A \circ \mathrm{ad}\, B)$ defines a natural energy function $\frac{1}{2}\int_0^T \langle U(t), U(t)\rangle dt$ associated with every horizontal curve $g(t)$ that is a solution of $\frac{dg}{dt} = g(t)(A + U(t))$, which then induces an optimal control problem of finding a horizontal curve that connects two given points in G along which the energy transfer is minimal. The solutions of this optimal control problem that will be the main focus of the lecture. In particular, I will show that the Hamiltonian for this problem, obtained by the Maximum Principle of optimal control, leads to the class of Hamiltonians on $\mathfrak{g}$ that admit spectral parameter representations with important contributions to

the theory of integrable Hamiltonian systems. Particular cases provide natural explanations for the classical results of Fock and Moser linking Kepler's problem to the geodesics on spaces of constant curvature, and J.Moser's work on integrability based on isospectral methods, in which C. Newmann's mechanical problem on the sphere and C. L. Jacobi's geodesic problem on an ellipsoid play the central role. The talk will also address the relevance of this class of Hamiltonians to the elastic curves on spaces of constant curvature.

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Ioan Marcut, Radboud Universiteit Nijmegen, The Netherlands

Title: A local model around Poisson submanifolds

Abstract: The first jet of a Poisson tensor at a fixed-point encodes precisely the structure of a Lie algebra (i.e. the isotropy Lie algebra). The corresponding linear Poisson structure on the dual of the isotropy Lie algebra (i.e. the Kirillov-Kostant-Souriau Poisson structure) represents the first jet approximation of the Poisson tensor. Much more intricate is the semi-global version of this construction due to Yuri Vorobiev, which provides a first order approximation for a Poisson structure around a symplectic leaf, depending only on the first order jet at the leaf (encoded by a transitive Lie algebroid). In this talk I will explain a similar model for first order approximations of Poisson structures around Poisson submanifolds. This model generalizes Vorobiev's construction, it depends only on the first jet of the Poisson structure, it is unique up to isomorphisms, but does not always exist. I will also discuss an existence criterion. This is joint work with Rui Loja Fernandes.

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Fatima Silva Leite, University Coimbra, Portugal

Title: Left-invariant sub-Riemannian systems on Lie groups and extremal curves on Stiefel manifolds

Abstract: This talk uncovers a large class of left-invariant sub-Riemannian systems on Lie groups that admit explicit solutions with certain properties and provides geometric origins for a class of interesting curves on Stiefel manifolds, called quasi-geodesics. These curves, which have constant geodesic curvature and project on Grassmann manifolds as Riemannian geodesics, have proved to be particularly important in solving interpolation problems arising in real applications. We also show that quasi-geodesics are the projections of sub-Riemannian geodesics generated by certain left-invariant distributions on Lie groups that act transitively on that manifold.

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Max Wardetzky, Georg-August-Universität Göttingen, Germany

Title: Discrete Curvature Functionals — and some of their challenges

Abstract: Discrete notions of curvatures are by now quite far developed, partially motivated by applications in shape-space analysis, computer graphics, and physical simulation. When establishing discrete notions of curvatures, one is often guided by mimicking structural properties of the classical smooth setting and by the desire to recover classical notions in the limit of refinement. While we have gained quite a bit of knowledge about the convergence of discrete curvatures and differential operators by now, one question remains widely open and poses interesting challenges: Do those minimizing shapes that we are able to compute approximate their smooth counterparts or do they just provide us with pretty pictures? Building on tools from variational analysis, I will present a framework for treating convergence of discrete minimizers of geometric energy functionals. As a particular example, I will discuss convergence of the perhaps most prominent minimizers of geometric energy functionals: discrete minimal surfaces.

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Stefan Waldmann & Knut Hüper