

Collected talks with abstracts of the Workshop

Math in the Mill 2019

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1 Friday, 26th April 2019

BENEDIKT HURLE (CHERN INSTITUTE OF MATHEMATICS, CHINA)

Deformation quantization of bimodules

26 April 2019

Abstract

In this talk I will first describe the general formal deformation of two algebras A , B and an $A - B$ -bimodule M . For this one can define a dgLa such that the Maurer-Cartan elements are given by an associative multiplication on A and B and a compatible bimodule structure on M . So the corresponding cohomology can be used to describe the construction of formal deformations. Then this will be applied to the situation where the algebras and module are smooth functions on different manifolds and the action is given as the pullback of a smooth map. In this situation the cohomology can often be computed if the maps are nice (the image is a submanifold). Finally I will give some remarks on the formality of the problem.

THOMAS BENDOKAT (UNIVERSITY OF SOUTHERN DENMARK, DENMARK)

Applied Differential Geometry and Manifold Interpolation

26 April 2019

Abstract

While many know differential geometry in the context of physics, it can also be applied to the real world. One case where differential geometry can be used is model reduction, for example when simulating airflow over an airplane. When problems are posed on a (Riemannian) manifold, one can exploit the geometry to, among other things, interpolate between sampled data points. But here computer constraints come into play: It is not enough to just have a function for computations, the function moreover needs to be computable in acceptable time. We take a look at Stiefel manifolds with the canonical metric, inherited from the quotient structure, and the associated Riemannian exponential and logarithm for the calculation of geodesics.

MARISA SCHULT (UNIVERSITY OF WÜRZBURG, GERMANY)

Life - As Told by Mathematicians

26 April 2019

Abstract

Some funny anecdotes and stories told by or about (in-)famous mathematicians.

2 Saturday, 27th April 2019

ANDREAS KRAFT (UNIVERSITÀ DEGLI STUDI DI SALERNO, ITALY)

Introduction to Hodge Theory

27 April 2019

Abstract

In this short talk we want to introduce and motivate the the classical Hodge theory of Riemannian and Kähler manifolds. At first, we recall the Hodge star operator, the codifferential and the definition of harmonic differential forms. After a really short digression to elliptic differential operators we state the main result: the Hodge decomposition theorem relating harmonic forms with the usual de Rham cohomology. If time permits, we finally mention some consequences and generalizations.

GREGOR SCHAUMANN (JULIUS-MAXIMILIANS-UNIVERSITÄT WÜRZBURG, GERMANY)

Introduction to Quantum invariants of knots

27 April 2019

Abstract

Knot theory is a classical branch of low-dimensional topology with a clear fundamental problem: When are two given knots "the same", i.e. isotopic? The main goal is thus to provide reasonable invariants that allow to distinguish different knots. After reviewing the most important classical knot invariants, we will investigate the quantum invariants: Those developed after Wittens QFT-interpretation of the Jones polynomial in the 90s and fit in the framework of topological quantum field theory.

MAXIMILAN HANUSCH (UNIV PADERBORN, GERMANY)

The Regularity Problem for Milnor's Infinite-Dimensional Lie Groups

27 April 2019

Abstract

In his famous survey article in 1984, Milnor formulated the concept of regularity for infinite-dimensional Lie groups (a la Bastiani) in order to enable proofs of fundamental Lie theoretic facts also in infinite dimensions. Roughly speaking, regularity is concerned with definedness and smoothness/continuity of the product integral – a notion that naturally generalizes the concept of the Riemann integral for curves in locally convex vector spaces to infinite dimensional Lie groups (Lie algebra valued curves are thus integrated to Lie group elements). For instance, (1) the exponential map of a Lie group is the restriction of the product integral to constant curves; and, (2) given principal fibre bundle, holonomies are product integrals of such Lie algebra valued curves that are pairings of smooth connections with derivatives of curves in the base manifold of the bundle. Although individual arguments show that all known example classes of infinite dimensional Lie groups admit regularity, only recently general regularity criteria had been found. In this talk, we present these results, including a complete solution to the differentiability (smoothness) issue, which forms a substantial part of the regularity problem. We furthermore discuss integrability conditions for Lie algebra valued curves; and show that C^0 -continuity of the product integral is equivalent to that the Lie group multiplication fulfills a generalized triangle inequality. As a real-life application, we discuss the strong Trotter property in this context.

LUKAS MIASKIWSKYI (DELFT UNIVERSITY OF TECHNOLOGY, NETHERLANDS)

Infinite-dimensional Symmetries and Jacobi Algebras

27 April 2019

Abstract

Incorporating symmetries of physical systems into their corresponding quantizations necessitates so-called projective representations, which are like usual representations "up to a phase". These kinds of representations are classified by the second group cohomology of the corresponding symmetry group. However, there are physically relevant symmetries which can only be described by Lie groups of infinite dimension. Hence, we investigate the broad, and still largely open question of how one can treat cohomology of infinite-dimensional Lie groups. After a short motivation, we present a rough overview of known results for some infinite-dimensional Lie groups and Lie algebras, and, some work-in-progress thoughts on so-called Jacobi algebras, an algebraic structure for certain Lie algebras of a geometric nature.

THORSTEN REICHERT (IBA DOSIMETRY)

Programming with Monoids

27 April 2019

Abstract

Especially in functional programming, certain algebraic structures have become well-established tools for problem solving. One of the most prominent ones are monoids, which are employed in myriad ways, with varying levels of sophistication and abstraction. Starting from the very basic versions and their applications to certain programming paradigms, this talk will, step-by-step, introduce more advanced monoids, which are then typically understood in the context of category theory.

3 Sunday, 28th April 2019

PHILIPP SCHMITT (UNIVERSITY OF COPENHAGEN, DENMARK)

Convergent star products

28 April 2019

Abstract

In this talk, I will first describe Rieffel's construction of deformed products for an algebra with an action of \mathbb{R}^d and explain why this is not sufficient to quantize the 2-sphere with its usual symplectic structure. I will then describe another approach developed by Beiser and Waldmann and a way to generalize it to coadjoint orbits of semisimple Lie groups.

MATTHIAS SCHÖTZ (UNIVERSITÉ LIBRE DE BRUXELLES, BELGIUM)

Su^* -algebras are the new C^* -algebras

28 April 2019

Abstract

C^* -Algebras are often used as models for observable algebras in classical and quantum physics. The main reasons for that are the Gelfand-Naimark representation theorems, their well-behaved spectral theory and their continuous calculus. However, the class of C^* -algebras is too small and does not include e.g. algebras with elements fulfilling the canonical commutation relations. Luckily, all the above properties of C^* -algebras can be generalized to certain classes of ordered * -algebras.