Julius-Maximilians-
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MUΦ
Lehrstuhl X

Collected talks with abstracts of the Workshop

BAWMAPF

Barcelona-Würzburg Mathematical Physics Factory

February 2021

Contents

1 Tuesday, 23 February 2021

Robert Cardona Aguilar (UPC Barcelona)

Local and semi-global action-angle coordinates in singular symplectic manifolds

23 February 2021

Abstract

In this talk, we will address the problem of local and global existence of action-angle coordinates for integrable systems in singular symplectic manifolds. Two types of singular geometric structures, whose singular locus is a hypersurface, are considered: b-symplectic manifolds [3] and folded symplectic manifolds [1]. In the b-symplectic case, integrable systems were already introduced and studied in [4]. For folded singularities, we introduce the analog of Hamiltonian dynamics and its corresponding notion of integrability. We will present some examples and prove an action-angle coordinate theorem around a regular fiber which lies in the singular hypersurface. Both kinds of integrable systems will be compared.

Going back to b-integrable systems, we construct examples in certain four dimensional b-symplectic manifolds and exhibit topological obstructions to the existence of semi-local action-angle coordinates. The obstructions arise from the topological monodromy that can exist in the singular locus of a b-symplectic manifold. This is joint work with Eva Miranda.

[1] A. Cannas da Silva, V. Guillemin, C. Woodward. *On the unfolding of folded symplectic structures.* Math. Res. Lett., 7(1):35-53, 2000.

[2] R. Cardona, E. Miranda. *Integrable systems on singular symplectic manifolds: From local to global.* Preprint (2020) arXiv:2007.10314.

[3] V. Guillemin, E. Miranda, A.R. Pires. *Symplectic and Poisson geometry on b-manifolds.* Adv. Math. 264 (2014), 864-896.

[4] A. Kiesenhofer, E. Miranda, G. Scott. *Action-angle variables and a KAM theorem for b-Poisson manifolds.* Journal des Mathématiques Pures et Appliquées, J. Math. Pures Appl. (9) 105 (2016), no. 1, 66-85.

Felix Menke (JMU Würzburg)

Coisotropic Algebras and a Coisotropic Serre-Swan Theorem

23 February 2021

Abstract

Looking at examples from Poisson geometry and from deformation quantization, we motivate the notion of coisotropic algebras. Essentially, those are triples of algebras which give a description of various reduction procedures in algebraic terms. When studying such triples, it makes sense to investigate also their associated category of coisotropic modules. In this talk we focus on *projective* coisotropic modules and aim for a coisotropic Serre-Swan Theorem: For examples of coisotropic algebras from differential geometry we want to relate their projective coisotropic modules to sections of vector bundle-like structures.

Anastasiia Matveeva (UPC Barcelona)

b^m-Symplectic Group Actions: Slices and Reduction

23 February 2021

Abstract

I will talk about group actions on b^m-symplectic manifolds which are Poisson manifolds that can be seen as symplectic with singularities. We prove the slice theorem and reduction for the group actions preserving such structure. In the talk I will focus on the examples of non-Hamiltonian actions coming from moduli spaces of flat connections and Yang-Mills equations. This is joining work with Eva Miranda.

2 Wednesday, 24 February 2021

Marvin Dippell (JMU Würzburg)

Deformation and Hochschild Cohomology of Coisotropic Algebras

24 February 2021

Abstract

Deformation quantization studies the quantization of classical mechanical systems by deforming the commutative product of the classical algebra of observables into a non-commutative one. Since the notion of coisotropic algebras gives an algebraic setting for various reduction procedures, both in classical and quantum mechanics, they can be used to study the behaviour of symmetry reduction in deformation quantization. To this end we will present the basics of the deformation theory of coisotropic algebras. In particular, we will show that obstructions to the deformation problem are encoded in a coisotropic analogue of Hochschild cohomology.

Pau Mir García (UPC Barcelona)

Geometric quantization via cotangent models

24 February 2021

Abstract

We work out cotangent lift models for integrable systems with non-degenerate singularities which can be of elliptic, hyperbolic and focus-focus type. Those singularities naturally appear in polarizations on compact manifolds given by integrable systems and, in particular, by semitoric systems. These structures also show up in algebraic geometry naturally, for instance in the K3 surface. When it comes to their quantization several models have been proposed, but none of them can compete with the model of Kähler quantization (which cannot always be applied in this case) in terms of independence of the polarization. We use different versions of the cotangent lift technique for different types of singularities (in the sense of Williamson) and we apply these models to define new geometric quantization for non-degenerate singularities. By complexifying the system, we obtain a unique cotangent model which allows us to unify the definitions. In contrast to the former works these models are finite dimensional on compact manifolds.

Michael Heins (JMU Würzburg)

Deformation Quantization: From Formal to Strict

24 February 2021

Abstract

Even a century after the advent of quantum theory, it is still not entirely clear how to pass from a classical Hamiltonian system to its appropriate quantum analogue. There are approaches of varying effectiveness such as the naive correspondence principle, Geometric Quantization and Deformation Quantization. In this talk, we start with an overview over the ideas and history of *formal* Deformation Quantization since its inception in 1978. The principal proposal is treating Planck's constant $\hbar$ as a formal parameter to construct another associative product $\star$ on the formal power series over the classical observable algebra. This allows to focus on the arising combinatorics required for associativity without having to worry about topology and analysis. *Strict* Deformation Quantization is then this passage back to a non-formal framework by means of honest convergence of the formal power series with respect to a suitable topology. We present a general scheme for constructing strict deformations, which has been proposed by Waldmann. Finally, we review how this has been achieved for some examples in recent years and give a first flavour of the mathematics that arose from it.