Collected talks with abstracts of the Workshop

Math in the Mill 2021

Contents

1 Thursday, 27th May 2021

Thomas Weber (University of Bologna)

Noncommutative differential geometry on Hopf algebras

27 May 2021

Abstract

In his celebrated article from 1989 Woronowicz introduced a covariant differential calculus on Hopf algebras, generalizing the Cartan calculus on Lie groups. The aim of this talk is to recall his construction and to explain how other notions of differential geometry, mainly connections, curvature and torsion, generalize to this noncommutative setting. On the way, we discuss the fundamental theorem of Hopf modules, which reveals that bicovariant bimodules form a braided monoidal category. If time permits, we formulate some open problems and hint at a few recent results of the speaker in collaboration with Paolo Aschieri.

Thomas Bendokat (University of Southern Denmark)

Model Order Reduction of Hamiltonian Systems

27 May 2021

Abstract

For our purposes, we define a Hamiltonian system to be a symplectic vector space together with a Hamiltonian function. The time evolution of such a Hamiltonian system is governed by the well known Hamilton's equations. In applications where a numerical solution of a (possibly high dimensional) Hamiltonian system is sought, it can be beneficial to project it onto a lower-dimensional symplectic subspace, and solve the reduced system there: A careful choice of this subspace can lead to good numerical approximations of the solution. Finding an optimal symplectic subspace however is not immediately solved, but can be stated as an optimization problem on the manifold of symplectic subspaces of fixed dimension. Furthermore, assume that the Hamiltonian system depends on a parameter. Then the optimal symplectic subspace also depends on this parameter, and if the optimal symplectic subspaces corresponding to some parameter values are known, an approximation of the optimal symplectic subspace corresponding to an unsampled parameter can be found via interpolation. To solve these problems, we take a Lie group based look at the manifold of symplectic subspaces and symplectic bases of these subspaces, termed symplectic Grassmann/Stiefel manifold, as well as numerical issues. Pseudo-Riemannian geodesics and so called retractions can then be used in the optimization and interpolation problems.

Lukas Miaskiwskyi (Delft University of Technology)

Cohomology Theories of Associative Algebras and Quantization of Gauge Symmetries

27 May 2021

Abstract

Every Chair-X person must one day face Hochschild cohomology, a space which is naturally assigned to any associative algebra and classifies its formal deformations. On the algebra of smooth

functions on a manifold, the famous Hochschild-Kostant-Rosenberg-Theorem tells us that this cohomology can reproduce both vector fields and differential forms, an indication that Hochschild cohomology may offer an algebraic perspective on geometry. In this talk, I want to expand this perspective by discussing cyclic cohomology, a modification of Hochschild cohomology which characterizes formal deformations of "closed" type, and can reproduce de Rham cohomology on the algebra of smooth functions. My main interest in this cohomology arises from its ability to describe continuous Lie algebra cohomology of gauge algebras, the space classifying infinitesimal quantizations of gauge symmetries. If time allows, we may learn something about topological tensor products on the way. This extends results by Janssens-Wockel (2013) and yields a continuous version of the Loday-Quillen-Tsygan-Theorem (1983, 1984).

This talk is based on joint work with Bas Janssens.

Tobias Schmude (University of Birmingham)

Why bother with Type Theory?

27 May 2021

Abstract

Type theory was conceived in the early 20th century in order to resolve the paradoxes of naive set theory. Although it didn't prevail as the most commonplace foundation of mathematics due to the development of axiomatic set theory, it later reemerged in many different shapes and forms to be applied in a variety of subjects such as theoretical computer science, linguistics, and, of course, the foundations of mathematics.

We will explore a certain kind of type theory, homotopy type theory (HoTT), which is a promising candidate as a system for everyday mathematics due to, for instance, the following reasons:

- In some aspects it's much closer to mathematical practice than set theory. In particular, isomorphic structures (groups, rings, modules etc.) are directly equal.
- It can be implemented in most standard computer proof assistants (since a lot of them are based on type theory anyway).
- As its name suggests, it has a connection to homotopy theory, providing a comparatively easy and natural approach to ∞-groupoids.

2 Friday, 28th May 2020

Marisa Schult (University of Adelaide)

The Selberg Trace Formula

28 May 2021

Abstract

This talk will introduce Selberg's Trace formula for a Laplace operator on a compact Riemann surface. Coming from linear algebra it makes sense to consider suitable generalisations of the trace, not just for matrices, but for operators. Exploring the basic example of a Laplacian on a (complex) compact connected one-dimensional manifold of genus greater or equal to two this reveals unexpected insides into the connection between the geometric structure of the surface and spectrum of the operator.

Jonas Schnitzer (University of Freiburg)

The homotopy theory of Maurer-Cartan elements in L_∞-algebras

28 May 2021

Abstract

The gauge equivalence of Maurer-Cartan elements of differential graded Lie algebras does not generalize to L_∞-algebras in a straight-forward way. In my talk, I will explain the notion of homotopy equivalence of Maurer-Cartan elements, which is a reasonable generalization of gauge equivalence. L_∞-morphisms between two fixed L_∞-algebras are Maurer-Cartan elements of the so-called convolution L_∞-algebra and hence we can speak of homotopy equivalence between them. On the other hand, one can twist L_∞-algebra structures and morphisms with Maurer-Cartan elements. As a last part I will explain how twisting with homotopy equivalent Maurer-Cartan elements and homotopy equivalence of twisted L_∞-morphisms interact.

Matthias Schötz (Université Libre de Bruxelles)

What is the Positivstellensatz and why should I care?

28 May 2021

Abstract

TBA

Michael Heins (University of Würzburg)

From Continuous to Entire Vectors

28 May 2021

Abstract

In this talk, we give an overview over various classical regularity notions for vectors with respect to a given representation on a sufficiently nice locally convex space. Starting with a strongly continuous Lie group action, we begin with a top down perspective to construct the corresponding Lie algebra action on a common dense domain. Indeed, we review a number of classical density results. Flipping the dynamic, we then assume a bottom up perspective and study integrability of Lie algebra representations to Lie group representations. Finally, the Taylor formula intertwines both pictures in a conceptual manner. Along the way, we discuss the various difficulties induced by the high degree of generality. We illustrate this by the surprisingly ill-behaved translation action of the Lie group G on its algebra of continuous functions $\mathscr{C}(G)$, which plays a prominent role in our recent research.

Maximilian Stegemeyer (Max Planck Institute for Mathematics in the Sciences)

Topological Invariants and Cut Loci

28 May 2021

Abstract

The cut locus of a point in a complete Riemannian manifold determines almost all of its topology. Studying the cut locus can however be very hard as there are in general not many techniques available for this matter. The so-called geodesic complexity, a numerical isometry-invariant of a Riemannian manifold, highly depends on the cut locus. This talk therefore outlines basic properties of the cut locus and shows how topological invariants can restrict the behaviour of the cut locus. In the end an outlook towards topological and geodesic complexity will be given.