Announcement

Seminar on Deformation Quantization

13. 5. 2022 at 2pm CEST

Hybrid Seminar in SE 30 and

`https://uni-wuerzburg.zoom.us/j/92529190594?pwd=WkJvR1o1QUdldUNSSjFJbHB4c0Z0dz09`

Matthias Schötz (JMU Würzburg)

Symmetry reduction of States and a non-commutative Positivstellensatz for $\mathbb{CP}^n$

On $\mathbb{CP}^n$ one can construct a (non-formal) star product e.g. by symmetry reduction out of the Wick star product on $\mathbb{C}^{1+n}$. This results in an $\hbar \in \mathbb{C} \setminus \Omega$-dependent associative product $\star_\hbar$ on the space of polynomials (in the sense of real algebraic geometry) on $\mathbb{CP}^n$ which deforms the pointwise product (Ω is discrete with sole accumulation point 0). For $\hbar \in \mathbb{R} \setminus \Omega$, the pointwise complex conjugation describes a *-involution on this algebra so that one obtains an $\hbar$-dependent family of *-algebras $\mathcal{P}_\hbar(\mathbb{CP}^n)$. One can then try to determine whether or not, or for which values of $\hbar$, these *-algebras $\mathcal{P}_\hbar(\mathbb{CP}^n)$ have non-trivial representations. An essentially equivalent problem is to classify all the algebraic states on $\mathcal{P}_\hbar(\mathbb{CP}^n)$, i.e. all linear functionals $\omega\colon \mathcal{P}_\hbar(\mathbb{CP}^n) \to \mathbb{C}$ that fulfil $\omega(a^* \star_\hbar a) \geq 0$ for all $a \in \mathcal{P}_\hbar(\mathbb{CP}^n)$ and $\omega(1) = 1$. But with the *-algebras $\mathcal{P}_\hbar(\mathbb{CP}^n)$ arising by symmetry reduction out of the well-understood Wick star product, one should rather ask: Under which conditions do algebraic states for the Wick star product on $\mathbb{C}^{1+n}$ descend to algebraic states on $\mathcal{P}_\hbar(\mathbb{CP}^n)$? Can all algebraic states on $\mathcal{P}_\hbar(\mathbb{CP}^n)$ be obtained this way? The latter question leads to a non-commutative Positivstellensatz for $\mathbb{CP}^n$, which, in contrast to its commutative analog, is non-strict.

Invited by Stefan Waldmann