Seminarankündigung

Deformationsquantisierung

Am 13. 6. 2019 spricht um 12 Uhr c.t.

Seminarraum SE 31.009

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Pointwise convergence of the integral kernels of Feynman path integrals

The Feynman path integral formulation of quantum mechanics is universally recognized as a milestone of modern theoretical physics. Roughly speaking, the core principle of this picture provides that the integral kernel of the time-evolution operator shall be expressed as a "sum over all possible histories of the system". This phrase entails a sort of integral on the infinite-dimensional space of suitable paths, to be interpreted in some sense as the limit of finite-dimensional short-time approximation operators. In spite of the suggestive heuristic arguments and the success as a practical tool for performing computations [4], the quest for a rigorous derivation of the Feynman path integrals is far from over. This is evidenced by the wide variety of attempts to give mathematical meaning to this framework, mainly with the equipment of functional, harmonic and stochastic analysis [1, 2].

Notwithstanding the several outcomes concerning the convergence in suitable operator topologies, the original Feynman's idea underlay the much more difficult and widely open problem of the pointwise convergence of the integral kernels of the approximation operators [3]. We address this problem and significantly benefit from concepts and techniques arising in the context of time-frequency analysis, which have been fruitfully applied to the study of path integrals only in recent times [5, 6, 7].

We consider path integrals in the Trotter-type form for the Schrödinger equation, where the Hamiltonian is the Weyl quantization of a real-valued quadratic form perturbed by a bounded non-smooth potential [8]. In a nutshell, we rephrase the problem in terms of pseudodifferential operators and then exploit the rich structure enjoyed by certain function spaces of a marked harmonic analysis flavour, defined in terms of the decay of the Fourier transform, namely the modulation spaces $M^\infty_s(\mathbb{R}^{2d})$ (with $s > 2d$) and $M^{\infty,1}(\mathbb{R}^{2d})$. In particular, they are Banach algebras for both pointwise multiplication and product of symbols for the Weyl calculus.

Joint work with Fabio Nicola.

1. S. Albeverio, R. Høegh-Krohn, and S. Mazzucchi: *Mathematical theory of Feynman path integrals. An Introduction*. Lecture Notes in Mathematics 523. Springer-Verlag, Berlin, 2008.
2. S. Albeverio, and S. Mazzucchi: Path integral: mathematical aspects. Scholarpedia, **6**(1):8832, 2011.
3. R. Feynman and A.R. Hibbs: *Quantum Mechanics and Path Integrals*. Emended Edition. Dover Publications, Mineola, 2005.

4. C. Grosche and F. Steiner: *Handbook of Feynman path integrals*. Springer, Berlin, 1998.
5. F. Nicola: Convergence in L^p for Feynman path integrals. *Adv. Math.* **294** (2016), 384–409.
6. F. Nicola: On the time slicing approximation of Feynman path integrals for non-smooth potentials. *J. Anal. Math.* **137**(2) (2019), 529–558.
7. F. Nicola, and S. I. Trapasso: Approximation of Feynman path integrals with non-smooth potentials. arXiv preprint, arXiv:1812.07487, 2018.
8. F. Nicola, and S. I. Trapasso: On the pointwise convergence of the integral kernels in the Feynman-Trotter formula. arXiv preprint, arXiv:1904.12531, 2019.

gez. Stefan Waldmann