Seminarankündigung

Deformationsquantisierung

Am 18. 12. 2020 spricht um 14 Uhr c.t.

`https://bbb.durates.net/b/ste-2va-uez`

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Multiplication of differential operators in terms of connections

There is a well-known explicit formula for the multiplication of two differential operators in any open set of $\mathbb{R}^n$ in terms of their symbols, by means of the global coordinates x and the additional 'conjugate' coordinates p. On a differentiable manifold equipped with a connection ∇ in the tangent bundle any differential operator can be parametrized by a symmetric tensor field paired with symmetrized iterated covariant derivatives (standard ordered or Lichnérowicz prescription, total symbol calculus). The product of two differential operators can also be written in this form, but the explicit form will contain complicated curvature and torsion terms which in general seem only to be known in terms of the (inverse of the)exponential map of ∇ and parallel transport. The problem is strongly related to the problem of finding explicit formulas for star-products on cotangent bundles of manifolds: these star-products had been treated long time ago by Fedosov, Bordemann/Neumaier/Waldmann, and Bordemann/Neumaier/Pflaum/Waldmann where existence and classification questions had been solved.

In this work we would like to express more explicitly the curvature and torsion terms appearing in the differential operator product. We have chosen the following algebraic approach which seem to work for general commutative rings K, A and any morphism $K \longrightarrow A$ as long K and hence A contains the rational numbers.

1. The Lie algebra of all vector fields on a manifold forms a Lie-Rinehart algebra L over the real commutative unital algebra A of all R-valued smooth functions on the manifold. The algebra of all differential operators is isomorphic to the so-called universal enveloping algebra $U(L, A)$ of L, hence we would like to describe these algebras in general. A-linearity (as opposed to just R-linearity) can be translated to geometry as 'fibrewise' or 'tensorial'.

2. Connections ∇ à la Koszul can be defined in this framework, as well as their iterations by copying the formulas from differential geometry. It turns out that unsymmetrized iterated covariant derivatives have very pleasant combinatorial properties, so a completely explicit formula can be obtained for symbols being smooth (not necessarily symmetric) tensor fields or elements of

$T_A(L)$ in algebraic terms. Here the A-linear cocommutative shuffle comultiplication turns out to be a very important piece of structure.

3. The desired enveloping algebra $U(L, A)$ will be a quotient of $T_A(L)$: the two-sided ideal $J(L, A)$ for the only R-linear multiplication (which we have to mod out) is also a coideal with respect to the A-linear comultiplication which can explicitly be described:

4. In the construction the primitive part of $T_A(L)$ (whose underlying A-module is the free A-Lie algebra generated by L) will become important: it is a Lie-Rinehart algebra over A isomorphic to M.Kapranov's path Lie algebroid (2007). There is a canonical morphism Z of Lie-Rinehart algebras form the primitive part to L whose kernel H carries a representation in L equal to the A-Lie algebra of infinitesimal holonomy. There is a recursion equation for Z in terms of curvature and torsion. The coideal $J(L, A)$ is generated A-linearly by the kernel H.

5. The remaining piece is the projection $T_A(L)$ to $U(L, A) \bmod J(L, A)$ which is a deformation of the usual projection describing the passage to the symmetric algebra $S_A(L)$. It can entirely expressed by rational combinatorics and the map Z which at least partially answers the above problem.

gez. Stefan Waldmann