Announcement

Seminar on Deformation Quantization

19. 11. 2021 at 2pm CET

Hybrid Seminar in SE 30 and

`https://uni-wuerzburg.zoom.us/j/92529190594?pwd=WkJvR1o1QUdldUNSSjFJbHB4c0Z0dz09`

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Modular Spectral Triples and deformed Fredholm modules

Due to possible applications to the attempt to provide a set of equations which unify the four elementary interactions in nature (the grand-unification) and in another, perhaps connected, direction in proving the long-standing, still unsolved, Riemann conjecture about the zeroes of the ζ-function, Connes' non-commutative geometry grew up rapidly in the last decades. Among the main objects introduced (by A. Connes) for handling noncommutative geometry there are the so called spectral triples, encoding most of the properties enjoyed by the (quantum) "manifold" into consideration, and the associated Fredholm modules. On the other hand, the so-called Tomita modular theory is nowadays assuming an increasingly relevant role for several applications in mathematics and in physics. Such a scenario suggests the necessary need to take the modular data into account in the investigation of quantum manifolds. In such a situation, the involved Dirac operators should be suitably deformed (by the use of the modular operator), and should come from non-type II_1 representations. Taking into account such comments, we discuss the preliminary necessary step consisting in the explicit construction of examples of non type II_1 representations and relative spectral triples, called modular. This is done for the noncommutative 2-torus $\mathbb{A}_\alpha$, provided α is a (special kind of) Liouville number, where the nontrivial modular structure plays a crucial role. For such representations, we briefly discuss the appropriate Fourier analysis, by proving the analogous of many of the classical known theorems in harmonic analysis such as the Riemann-Lebesgue lemma, the Hausdorff-Young theorem, and the L^p-convergence results associated to the Cesaro means (i.e. the Fejer theorem) and the Abel means reproducing the Poisson kernel. We show how those Fourier transforms "diagonalise" appropriately some examples of the Dirac operators associated to the previous mentioned spectral triples. Finally, we provide a definition of a deformed generalisation of "Fredholm module", i.e. a suitably deformed commutator of the "phase" of the involved Dirac operator with elements of a subset (the so-called Lipschitz $*$-algebra or Lipschitz operator system) which, depending on the cases under consideration, is either a dense $*$-algebra or an essential operator system. We also show that all models of modular spectral triples for the noncommutative 2-torus mentioned above enjoy the property to being also a

deformed Fredholm module. This definition of deformed Fredholm module is new even in the usual cases associated to a trace, and could provide other, hopefully interesting, applications. The present talk is based on the following papers:

[1] F. Fidaleo and L. Suriano: *Type III representations and modular spectral triples for the noncommutative torus*, J. Funct. Anal. **275** (2018), 1484- 1531.

[2] F. Fidaleo: *Fourier analysis for type III representations of the noncom- mutative torus*, J. Fourier Anal. Appl. **25** (201), 2801-2835.

[3] F. Ciolli and F. Fidaleo: *Type III modular spectral triples and deformed Fredholm modules*, preprint.

Invited by Stefan Waldmann