

Im Oberseminar

Deformationsquantisierung

spricht am 27.05.2016 um 14 Uhr c.t.,

im Seminarraum 00.009 (Physik Ost)

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über das Thema:

An unusual power series expansion for certain holomorphic functions

Let $\overline{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ be the Riemann-sphere,

$$\Omega \,:=\, \left(\,\overline{\mathbb{C}}\times\overline{\mathbb{C}}\,\right)\,\backslash\,\left(\,\left\{\,(x,y)\in\mathbb{C}\times\mathbb{C}\,\,\middle|\,\, xy=1\,\right\}\cup\left\{(0,\infty),(\infty,0)\right\}\,\right)$$

and denote by $\mathcal{O}(\Omega)$ the Fréchet space of all holomorphic functions on Ω with the usual topology of uniform convergence on all compact subsets of Ω . I will prove (or sketch the prove) that all $\hat{f} \in \mathcal{O}(\Omega)$ can be represented by a locally-uniformly and absolutely converging power-series

$$\hat{f}(x,y) = \sum_{p,q=0}^{\infty} f_{p,q} \,\hat{e}_{p,q}(x,y) \qquad \text{for all } (x,y) \in \Omega$$
 (*)

with
$$\hat{e}_{p,q}(x,y) \in \mathcal{O}(\Omega)$$
 given by $\hat{e}_{p,q}(x,y) = \frac{x^p y^q}{(1-xy)^{\max\{p,q\}}}$ for all $(x,y) \in \Omega \cap \mathbb{C}^2$.

More precisely, let $\mathcal{A} \subseteq \mathbb{C}^{\mathbb{N}_0 \times \mathbb{N}_0}$ be the subspace of all series fulfilling $||f||_R < \infty$ for all $R \in \mathbb{R}^+$, where

$$A \ni f \mapsto ||f||_R := \sum_{p,q=0}^{\infty} |f_{p,q}| R^{p+q} \in [0,\infty],$$

and endow \mathcal{A} with the locally convex topology of all these seminorms $||\cdot||_R$. Then mapping a series $f \in \mathcal{A}_0$ to $\hat{f} \in \mathcal{O}(\Omega)$ like in (*) is an isomorphism of Fréchet spaces.