Julius-Maximilians-
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MUΦX Lehrstuhl

Announcement

Seminar on Deformation Quantization

29. 3. 2022 at 10am CET

Hybrid Seminar in SE 31 and

https://uni-wuerzburg.zoom.us/j/92529190594?pwd=WkJvR1o1QUdldUNSSjFJbHB4c0Z0dz09

Martin Bordemann (UHA Mulhouse)

Drinfel'd associators for the working algebraist refusing to use more than second year university analysis I

The content of the talk is more or less well-known and is focused on an 'elementary' proof of the hexagon and pentagon identities for the famous Drinfel'd associator (V.G.Drinfel'd, 1989/90). The latter is a famous universal formal series in noncommutative monomials of two elements of any complex associative algebra. It applications range from number theory (the coefficients in the series are related to multiple zeta values), to quantization of Lie bialgebras (P.I.Etingof and D.A.Kazhdan, 1996), dequantization of bialgebras and deformation quantization (D.E.Tamarkin, 1999), and quantization of Lie quasibialgebras (B.Enriquez, G.Halbout, 2008). The definition of the associator, and the proof of the hexagon (in any algebra) and pentagon identities (in the Drinfel'd-Kohno algebra) seem to be not quite easy to grasp by the average mortal mathematician, and the original articles and the usual text books on the matter stay somewhat fuzzy about it. The main idea –as in many 'Russian' topics of mathematical physics– is 'monodromy', i.e. to prove identities in certain algebras by representing them as parallel transports over two different composed paths joining the same pair of points in a simply connected domain with respect to some (formally) flat connection (e.g. the one by V.G.Knizhnik, A.B.Zamolodchikov, 1984). The main difficulty is the use of general solutions in terms of 'multi-valued functions' of the linear systems which are only local or global on universal covering spaces (multi-valued functions) which can be inaccessible. We shall just use limits of parallel transports.

My talk presents work in collaboration with Andrea Rivezzi and Thomas Weigel from the University of Milano-Bicocca. In the first talk there will be and introduction about associators and the above-mentioned relations. I shall then review the necessary analysis (hopefully up to second year) which is formal linear differential equations in terms of iterated integrals, then formal connections and parallel transports (where a certain reparametrization invariance plays a crucial role), and some remarks on the norms we shall be using (in fact for each order in the formal parameter we deal with a finite-dimensional complex vector space). At the end of part 1 I hope to introduce the associator as an elementary parallel transport on the interval where the behaviour at the singularities at 0 and 1 can be factorized off while choosing families of paths parametrized by strictly positive real number δ (which in the end is send to 0).

Part two concerns some properties of the Drinfel'd associator (multiple zeta values) including the hexagon equations which can be derived by parallel transports along simple paths (depending on δ) in the doubly punctured complex plane $\mathbb{C}$. The proof of the pentagon equation has been inspired by a picture in C.Kassel's book 'Quantum Groups', 1995, p.478, which we have translated into concrete paths (depending on δ) in the open subset of $\mathbb{R}^4$ consisting of the points $x_1 < x_2 < x_3 < x_4$. In general, each parallel transport can be factorized into a singular (for $\delta \to 0$) term, a wanted term and a complicated, but harmless term, and in the identities the singular terms cancel, and the complicated harmless terms go 1 (for $\delta \to 0$).

Invited by Stefan Waldmann