

Seminar on Deformation Quantization and Geometry

20. 10. 2023 at 2pm CEST/CET $\,$

Seminarroom SE 30 $\,$

MATTHIAS FRERICHS (JMU WÜRZBURG)

Grothendieck-Rings of Grothendieck Verdier Categories

We motivate Grothendieck-Verdier duality on a category by considering categories of bimodules $\mathcal{A} - \mathcal{A} - \mathsf{Bimod}$ over an algebra \mathcal{A} . Here the well known notion of rigid duality can be shown to be insufficient and instead of a representation of $\operatorname{Hom}_{\mathcal{A},\mathcal{A}}(-\otimes_{\mathcal{A}} N,\mathcal{A})$ for the monoidal unit \mathcal{A} and a bimodule $\mathcal{A} - \mathcal{A} - \mathsf{Bimod}$ as we would expect for a rigid category, we obtain one for $\operatorname{Hom}_{\mathcal{A},\mathcal{A}}(-\otimes_{\mathcal{A}} N,\mathcal{A}^*)$ for an algebra $\mathcal{A}^* \neq \mathcal{A}$.

In general a Grothendieck-Verdier structure on a category \mathfrak{C} yields a dualizing functor $\mathsf{D} \colon \mathfrak{C} \to \mathfrak{C}$ which represents $\operatorname{Hom}_{\mathfrak{C}}(-\otimes X, K)$ for some fixed object $K \in \mathfrak{C}$. This induces a map on the Grothendieck-Ring $\operatorname{Gr}(\mathfrak{C})$.

We consider the following questions:

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- What algebraic structure does a Grothendieck-Verdier duality induce on the Grothendieck-Ring?
- Given a Grothendieck-Ring with some algebraic structure and a categorification of it, when does the categorification inherit a Grothendieck-Verdier structure?

We answer both questions in the case of semisimple categories by defining the structure of a gvbased ring and give some examples of such a ring structure as well as categorifications obtained from quivers.

Invited by Stefan Waldmann

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