Announcement

Seminar on Deformation Quantization and Geometry

26. 4. 2024 at 14:00 s.t.

Seminarroom SE 30

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From Homological Algebra to the Derived Category

The basic idea of homological algebra is to replace an object X in an abelian category $\mathcal{C}$ by a resolution $X_\bullet$, which represents a generalised construction of the object. Having picked such a resolution $X_\bullet$ one then can wonder whether operations between objects X, Y, formalised by functors $F : X \to Y$, respect their resolutions $X_\bullet, Y_\bullet$. This leads to the theory of derived functors $\mathrm{R}^j F$, which measure the failure of the given operation F to respect these generalised constructions. Using these derived functors cohomology theories known from different parts of mathematics can be formulated, like Lie algebra cohomology or sheaf cohomology.

However, when passing to the derived functors one is left with only the cohomology groups thus losing information about the objects and their resolutions. The derived category $\mathcal{D}(\mathcal{C})$, introduced by Grothendieck and Verdier, is an attempt to instead work with the resolutions directly. This derived category $\mathcal{D}(\mathcal{C})$ is a powerful invariant of the original category $\mathcal{C}$, having again interpretations in different parts of mathematics.

In this talk I will try to introduce the fundamental ideas of homological algebra with the goal of understanding the idea behind the construction of the derived category.

Invited by Stefan Waldmann