Program of the Workshop

Math in the Mill 2025

16. – 18. May 2025

	Friday, 16.05.	**Saturday, 17.05.**	**Sunday, 18.05.**
8:00		Breakfast	Breakfast
9:00		Niklas Orf	Matthias Frerichs
10:00		Chiara Esposito	Michael Heins
11:00	Arrival	Jonas Schnitzer	
12:00	Lunch	Lunch	Clean-Up
13:00			
14:00	Matthias Schötz	Francesco Cataffi	Departure
15:00	Thomas Weber	Christiaan van de Ven	
16:00	Marvin Dippell	Thorsten Reichert	
17:00			
18:00	Dinner	Pizza	
21:00	Minona Schäfer		

Matthias Schötz (University of Ljubljana)

Real Nullstellensatz for Lie algebras

Abstract

The commutative real Nullstellensatz states that an ideal I of a polynomial algebra in n variables is the vanishing ideal of a subset of R^n if and only if I is real, meaning that whenever $a_1^2 + ... + a_k^2$ is in I, then $a_1, ..., a_k$ individually are in I. There are similar results from real algebraic geometry that have been generalized to the noncommutative setting, replacing commutative real algebras by $*$-algebras and evaluation at points by (irreducible) $*$-representations. However, the search for noncommutative real Nullstellensätze has not been so successful yet. In this talk I will discuss what a hypothetical real Nullstellensatz for Lie algebras could look like, give a glimpse at the proof for 2-step nilpotent real Lie algebras (joint work with Philipp Schmitt) and what hurdles have to be overcome for more general classes of Lie algebras.

Thomas Weber (University of Prague)

Hopf algebras and quantum principal bundles

Abstract

I will give a gentle introduction to the theory of Hopf–Galois extensions and explain their geometric interpretation as noncommutative principal bundles. If time permits, I will review a differential calculus approach for quantum principal bundles, which gives rise to horizontal and vertical forms, together with a noncommutative Atiyah sequence. Examples, such as the quantum Hopf fibration, will be discussed.

Marvin Dippel (University of Salerno)

Classification of Homogeneoues Star Products

Abstract

This talk will mainly be a gentle introduction to homogeneous star products on the total space of a vector bundle. I will emphasize how a deformation retract for Hochschild cohomology simplifies many aspects of the theory of homogeneous star products. If time permits, we will discuss a classification of homogeneous star products by the second Lie algebroid cohomology.

Minona Schäfer (University of Würzburg)

Neo-Riemannian Theory

Abstract

What do music and group theory have to do with each other? And how can tones be represented on a torus? Neo-Riemannian theory provides an explanation by characterizing the relationships between triads through specific group actions. In this talk, we will introduce harmonic analysis methods from Viennese and German theory, thereby motivating the fundamental concepts of Neo-Riemannian theory. Moreover, we will demonstrate a mathematical application of the Eulerian Tonnetz for composing four-part chord progressions. So listen closely!

Niklas Orf (University of Würzburg)

Dirac Structures and their Integration

Abstract

Dirac structures – certain subbundles in the generalized tangent bundle ($\mathbb{T}M$) of a manifold, introduced by Courant in the 1990s – usually come up in the context of constrained mechanics, generalizing the Poisson geometric formulation. In this talk we will give a brief overview of these objects; in particular, we study the integration theory of their associated Lie algebroid. The corresponding class of groupoids was first described by Bursztyn, Crainic, Weinstein, and Zhu. Based on that, we take a somewhat simplified approach to get a glimpse of this correspondence.

Chiara Esposito (University of Salerno) & Jonas Schnitzer (University of Pavia)

Equivariant formality and reduction

Abstract

In this talk we will present a recent work of us, joint with R. Nest and B. Tsygan, in which we prove the existence and classification of the quantum momentum map. Time permitting, we will also explain what are the open questions concerning the quantization/reduction diagram and a chain version of equivariant formality.

Francesco Cattafi (University of Würzburg)

Pseudogroups and geometric structures

Abstract

The space of (local) symmetries of a given geometric structure has the natural structure of a Lie (pseudo)group. Conversely, geometric structures admitting a local model can be described via the pseudogroup of symmetries of such local model.

This philosophy can be made precise at various levels of generality (depending on the definition of "geometric structure") and using different tools/methods. In this talk I will present some aspects of a new framework, which includes previous formalisms (e.g. G-structures or Cartan geometries) and allows us to prove integrability theorems.

A main novelty of this point of view consists of the fact that it uncovers the (beautiful!) hidden structures behind Lie pseudogroups and geometric structures. Indeed, the relevant objects which make this approach work are Lie groupoids endowed with a multiplicative "PDE-structure", their principal actions, and the related Morita theory. Poisson geometry provides the guiding principle to understand those objects, which are directly inspired from, respectively, symplectic groupoids, principal Hamiltonian bundles, and symplectic Morita equivalence.

This is based on a forthcoming book written jointly with Luca Accornero, Marius Crainic and María Amelia Salazar.

Christiaan van de Ven (University of Erlangen)

Large deviations in the semi-classical limit of quantum spin systems

Abstract

The continuous C^*-bundle generated by Berezin-Toeplitz quantization on a symplectic manifold provides a rigorous framework for describing the semiclassical limit of quantum systems. The rate of convergence in this limit, often quantified by a rate function or entropy, is naturally formalized through the theory of large deviations. In this work, we investigate the specific case of the complex projective line, which serves as the phase space for a single quantum spin system. The spin, which characterizes the system's quantum nature, defines the semiclassical parameter and corresponds to the dimension of the associated Hilbert space. More precisely, we establish a full large deviation principle for the local Gibbs state and explicitly characterize the corresponding rate function. This is done by using a sophisticated argument based on the Feynman-Kac integral.

Thorsten Reichert

Architecture, Philosophy and fire-breathing Lizards: Intro to Residuality Theory

Abstract

One of the central issues in software architecture is dealing with time, uncertainty and change. Or in other words, to build rigid (software) systems, that survive in an ever-changing environment. Some fresh ideas to help with these issues come, surprisingly, from philosophy and have been mashed together with complexity theory into a new approach to software architecture, called Residuality Theory, by Barry O'Reilly. This talk will outline the core problems and the central ideas of Residuality Theory, including the (pretty fun and lizard-y) way of actually creating software architectures.

Matthias Frerichs (University of Greifswald)

Coarse Geometric Approach to Topological Phases of Matter

Abstract

Invariants of topological phases of matter used in solid-state physics are often extracted by considering topological invariants of the Bloch bundle, for example, via its Chern number in dimension 2 to determine Chern insulators. This, however, conceptually depends on the periodicity of the underlying lattice of atoms, which in reality cannot be guaranteed and is completely absent in amorphous materials.

I provide a brief introduction to determining such invariants directly from the Hamiltonian by examining the KK-theory of an associated Roe algebra. In combination with coarse cohomology classes of the underlying set of atoms, this approach should replicate the classical topological invariants determined via the Bloch bundle.

Finally, we discuss the determination of phases, i.e., homotopy classes of quantum systems. This is generally not straightforward, so we consider obstructions to such homotopies given by Berry classes and generalize using coarse geometry.

Michael Heins (University of Salerno)

Embracing monodromy: The orbit knows the way

Abstract

In this talk, we take a closer look at integrability and complexification of Lie group and algebra representations on locally convex spaces. The guiding example consists in the translation action of a Lie group on its algebra of continuous functions. Restricting ourselves to the smooth category, its infinitesimal counterpart reveals itself as Lie differentiation in direction of left invariant vector fields. On the further subalgebra of analytic mappings, the Lie-Taylor formula encodes the integration of the Lie algebraic action back to the original group action. This, in turn, naturally leads towards holomorphic functions on the complexified group. Taking a step back, demanding the same regularity for the orbit mappings

$$\pi_v \colon G \longrightarrow V, \quad \pi_v(g) \coloneqq \pi(g)v$$

of a representation π on a locally convex space V defines distinguished subspaces of V. Combining these notions of regularity for vectors with Lie's Theorems and the Monodromy Theorem facilitates differentiation, integration and complexification of various flavours of representations. The obstructions then become the non-triviality of the corresponding space. Closing the circle – and causing further monodromy – we prove the universality of our guiding example.