Preliminary Schedule for the Workshop

Poisson Geometry, Higher Structures, and Deformation Theory

Gregor Schaumann, Stefan Waldmann

University of Würzburg, 20. – 22. September 2023

The schedule: overview

Time	Wednesday	Time	Thursday	Time	Friday
10.00	Gutt	9.00	Fernandez Alvarez	9.00	Scheimbauer
11.00	Coffee	10.00	Coffee	10.00	Coffee
11.30	Jotz	11.00	Miti	11.00	Blohmann
12.30	Lunch	12.00	Lunch	12.00	Lunch
14.00	Bordemann	13.30	Cabrera	13.30	Bursztyn
15.00	Coffee	14.30	Coffee	14.30	Coffee
15.30	Janssens	15.00	Schnitzer	15.00	Esposito
18.00	Beer and Brezeln	19.00	Alte Mainbrücke		

Christian Blohmann

Hamiltonian Lie algebroids and Poisson reduction

Abstract

Hamiltonian Lie algebroids generalize the Lie algebroids of hamiltonian Lie algebra actions, both over presymplectic and Poisson manifolds. A hamiltonian structure on a Lie algebroid over a Poisson manifold consists of a momentum section of the dual bundle of the Lie algebroid together with a linear connection. The axioms generalize those of a hamiltonian action. I will show that the zero locus of the momentum section is coisotropic and invariant. The coisotropic distribution is dual to the image of the anchor. This means that the usual method of Poisson reduction can be applied, assuming suitable smoothness conditions. Finally, I will give a simple example of a non-hamiltonian Lie algebra action with a hamiltonian action Lie algebroid.

Martin Bordemann

P. Ševera's proof of the quantization of Lie bialgebras

Abstract

We shall report on P.Ševera's work in Sel. Math. 22 (2016) on the quantization of Lie bialgebras: the problem posed by V.G.Drinfel'd at the end of the eighties had been solved by P.I.Etingof and D.A.Kazhdan 1996. It turns out to be important (in the differential graded version) for D.E.Tamarkin's proof (1999) of M.L.Kontsevich's formality theorem in deformation quantization (1997). The proof by Etingof-Kazhdan requires a sophisticated topological treatment of dual spaces of infinite-dimensional vector spaces ('equicontinuous modules') which is not very easy to access. In contrast, Ševera's proof avoids dual spaces and topological arguments, and is much easier to unzip. His proof, as the one by Etingof-Kazhdan, uses Drinfel'd associators, Lie theory and diagram chase in braided monoidal categories. We shall also comment on the dequantization functor (B.Enriquez, P.I.Etingof 2005) requiring PROPs. Joint with Andrea Rivezzi and Th.Weigel.

Henrique Bursztyn

Graded geometry and generalized reduction

Abstract

We will discuss reduction procedures in generalized geometry (for Courant, Dirac and generalized complex structures). Our constructions rely on the graded symplectic viewpoint to Courant algebroids, and coisotropic and hamiltonian reductions in the graded setting. This is based on joint work with Cattaneo, Mehta and Zambon.

Alejandro Cabrera

Non-formal quantization of Poisson manifolds and Integrability

Abstract

In this talk, I will report on work in progress with R.L. Fernandes about the relation between certain non-formal families of star products quantizing a given Poisson manifold M and the problem of integrability of M to a (global) symplectic groupoid. First, we shall introduce the relevant families based on semiclassical Fourier integral operators and see that they induce a local symplectic groupoid G integrating M. The main result (in progress) is that the obstructions to "globalization" of G at the Poisson-geometric level necessarily translate into a lack of associativity for long words of elements in the (partial) star algebra. In particular, this identifies the integrability of M as an obstruction to admitting certain strict deformation quantizations as well as faithful representations of this type of star products.

Chiara Esposito

Deformation and Hochschild Cohomology of Coisotropic Algebras

Abstract

In this talk we will introduce the notion of coisotropic algebras, which consists of triples of algebras for which a reduction can be defined and unify in a completely algebraic fashion coisotropic reduction in several settings. Thus, we will discuss the theory of (formal) deformation of coisotropic algebras and show that deformations are governed by suitable coisotropic DGLAs. The obstructions to existence and uniqueness of formal deformations of coisotropic algebras are described via the Hochschild cohomology associated to coisotropic DGLAs. Finally, we will present some geometric examples. This is a joint work with Marvin Dippell and Stefan Waldmann (University of Wuerzburg).

David Fernandez Alvarez

Noncommutative Poisson geometry and pre-Calabi-Yau algebras

Abstract

A long-standing problem in Poisson geometry has been the definition of suitable "noncommutative Poisson structures". To solve it, M. Van den Bergh introduced double Poisson algebras and double quasi-Poisson algebras, which can be regarded as noncommutative analogues of the usual Poisson and quasi-Poisson manifolds, respectively. N. Iyudu and M. Kontsevich found an insightful correspondence between double Poisson algebras and pre-Calabi-Yau algebras; certain cyclic A_∞-algebras which can be seen as noncommutative versions of shifted Poisson manifolds. In this talk I will present an extension of the Iyudu-Kontsevich correspondence to the differential graded setting. I will also explain how double quasi-Poisson algebras give rise to pre-Calabi-Yau algebras. Interestingly, they involve an infinite number of non-vanishing higher multiplications weighted by the Bernoulli numbers. This is a joint work with E. Herscovich (Grenoble).

Simone Gutt

Around almost complex structures

Abstract

Smooth almost complex structures on manifolds (in particular on symplectic manifolds) have various integrability properties. We have been interested in defining relevant properties which a non integrable almost complex structure may have, in terms of its Nijenhuis tensor. In particular, we define the notions of minimally or maximally non integrable almost complex structures, and the notion of transverse complex structure defined by an almost complex structure. We review some Dolbeault-type cohomologies associated to an almost complex structure.

Bas Janssens

Central extensions from multisymplectic geometry

Abstract

In much the same way as the Lie algebra of hamiltonian vector fields is covered by the Poisson algebra in symplectic geometry, it is covered by the Lie n-algebra of observables in n-plectic geometry. Truncating this Lie n-algebra results in a central extension of the Lie algebra of hamiltonian vector fields by the de Rham cohomology in degree n-1. We show that for volume forms (where n is as high as possible), this central extension is in fact universal in the category of locally convex topological Lie algebras. This is obviously not the case for symplectic forms (where n is as low as possible), but here one can play a similar game with a different L_∞-algebra, one that naturally lives on the Poisson cohomology complex. Finally, we show that a smooth

hamiltonian action of a (not necessarily finite dimensional) connected Lie group G on an integral 2-plectic manifold gives rise to smooth central extension of G by $H^1(M, U(1))$, a higher analogue of the KKS extension. Combined with the aforementioned result, the yields a universal central extension for the group of volume-preserving diffeomorphisms of a compact 3-manifold. This is joint work with Cornelia Vizman, Leonid Ryvkin and Peter Kristel.

Madeleine Jotz

A geometrisation of positively graded manifolds

Abstract

Lie 2-algebroids are geometrised by linear Courant algebroids, while symplectic Lie 2-algebroids correspond to mere Courant algebroids. This talk begins by explaining these correspondences due to Li-Bland, Severa and Roytenberg, by establishing the underlying equivalence between [2]-manifolds and metric double vector bundles. The latter yields a dictionary between graded geometric structures on [2]-manifolds, like homological vector fields, Poisson and symplectic structures, and corresponding 'classical geometric' structures on the corresponding metric double vector bundles.

Metric double vector bundles dualise to double vector bundles equipped with a (signed) involution. The latter can then be understood as S_2-symmetric double vector bundles – recovering Pradines' 'inverse' symmetric double vector bundles.

Similarly, positively graded manifolds of arbitrary degree n are equivalent to n-fold vector bundles equipped with a (signed) S_n-symmetry. This talk explains this correspondence, and how symmetric multiple vector bundles, which are indexed by cube categories, provide a new and insightful point of view on (positively) graded geometry.

This work is partly joint with Malte Heuer, and partly joint with Leonid Ryvkin.

Antonio Miti

Multisymplectic observables and higher Courant algebroids

Abstract

Multisimplectic manifolds are a straightforward generalization of symplectic manifolds where one considers closed non-degenerate k-forms in place of 2-forms. Recent works by Rogers and Zambon showed how one could associate such a geometric structure with two higher algebraic structures: an L_∞-algebra of observables and an L_∞- algebra of sections of the higher Courant algebroid twisted by the multisymplectic form ω. The scope of this talk is to report on joint work with Marco Zambon (arXiv:2209.05836).

Our main result is proving the existence of an L_∞-embedding between the above two L_∞-algebras, generalizing a construction already found by Rogers around 2012 for multisymplectic 3-forms only. Moreover, we display explicit formulae for the sought morphism involving the Bernoulli numbers.

Although this construction is essentially algebraic, it also admits a geometric interpretation when declined to the particular case of pre-quantizable symplectic forms. Moreover, the latter case provides some evidence that this construction may be related to the higher analog of geometric quantization for integral multisymplectic forms.

Claudia Scheimbauer

Factorization algebras: an algebraic tool in field theories and higher algebra

Abstract

Factorization algebras were originally developped in the context of observables of a field theory and the structure thereof when quantizing. We will start with an introduction to the notion of a factorization algebra and a slight modification thereof. Then we explain some examples coming from algebras and modules. I will explain several results using factorization algebras: first, we discuss a conjecture by Lurie about dualizability whose proof is work in progress with Eilind Karlsson. Then I will explain how to use factorization algebras to describe a variant of "non-abelian" chain complexes. Examples of the latter arise from a loop construction from filtered spaces, and adding extra assumption, we can recover the space from its image. This is a relative version of May's recognition principle and is work-in-progress with Grég Ginot and Tashi Walde.

Jonas Schnitzer

Deformations of Lagrangian Q-submanifolds

Abstract

$\mathbb{N}$-graded symplectic Q-manifolds encompass a lot of well-known mathematical structures, such as Poisson manifolds, Courant algebroids, etc. Their Lagrangian Q-submanifolds are of special interest, since they simultaniously generalize coisotropic submanifolds, Dirac-structures and also serve as boundary conditions in AKSZ sigma models. In this talk we set up their deformation theory inside a symplectic Q-manifold via strong homotopy Lie algebras, which generalizes knows results including the deformation theory of coisotropic submanifolds and Dirac structures. This is a joint work with Miquel Cueca.