Julius-Maximilians-
UNIVERSITÄT
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MUΦX
Lehrstuhl

Collected talks with abstracts of the Seminar

# Deformation Quantization

of the Chair for Mathematical Physics
Institute of Mathematics
University Würzburg

## Contents

# 1 Summer Term 2023

Paul Beyer (JMU Würzburg)

## The Search for a Kähler Structure on the Cotangent Bundle of a Lie Group

**21. 7. 2023**, 3.30 pm CEST

### Abstract

In a first part we want to show how the well-regarded construction of a Wick star product by Fedosov can be achieved on a Kähler manifold as well. This product is then compatible with the complex structure of the Kähler manifold and the left- and right-multiplication with holomorphic and anti-holomorphic maps becomes locally the pointwise product. Furthermore, the usual way of translating between the Wick and Weyl product can be achieved here explicitly.

After this motivational part I will report about what became the main quest of my Master's Thesis: Applying this construction to the cotangent bundle of a Lie group. For this, off course, one needs a Kähler structure. The process of finding one turned out to be not as easy as expected, unfortunately. The first intuitive tries did not work out and it took some time to notice that there is a diffeomorphism between the cotangent bundle and the complexification of the underlying Lie group. The problem was that the pullback of the canonical complex structure of the complexification actually yields a different Lie group structure than what we were expecting.

Christopher Rudolph

## The Recuction Scheme for Star Products on Cotangent Bundles by Niels Kowalzig, Nikolai Neumaier and Markus J. Pflaum

**21. 7. 2023**, 2pm CEST

### Abstract

In this talk we recall a method to construct star products on Marsden-Weinstein reductions of classical phase spaces introduced by Kowalzig, Neumaier and Pflaum in 2005. Starting with a smooth, proper and free action $\Phi$ of a Lie group $G$ on a smooth manifold $Q$ one considers the case, where the reduction of $T^*Q$ with respect to the action given by the cotangent lift of $\Phi$ is symplectomorphic to $\big(T^*(Q/G), \omega_0\big)$. Viewing $Q$ as a principal $G$-bundle over $Q/G$ and choosing a certain connection one-form one obtains an isomorphism between the formal power series of smooth functions on $T^*(Q/G)$ polynomial in the momenta and the formal power series of horizontal $G$-invariant polynomial functions on $T^*Q$. In particular, given a star product on $T^*Q$ its projection on formal power series of horizontal $G$-invariant polynomial functions induces a star product on $T^*(Q/G)$ via pull-back.

Alcides Buss (Prodi professor, JMU)

## A new approach to Rieffel Deformation of C*-algebras via coactions of groups

**14. 7. 2023**, 2pm CEST

### Abstract

In this talk we revisit the procedure of deforming coactions of locally compact groups on $C^*$-algebras via Borel 2-cocycles on the underlying group $G$. This has been done before in an ad hoc way by Bhowmick, Neshveyev, and Sangha for reduced (or normal) coactions of groups, that is, coactions that belong to the reduced group $C^*$-algebra of $G$. Our new approach gives a systematic way to deform not only reduced, but also maximal and other exotic forms of $G$-coactions. The main idea is to use exotic versions of Landstad duality for coactions of $G$. This works for any crossed-product functor for $G$ that respects Morita equivalences. Our approach yields a new realization of the deformed $C^*$-algebras and many of the previous results carry over to this more general setting, including invariance of $K$-theory for deformations via homotopic 2-cocycles.

This is based on joint work with Siegfried Echterhoff.

JOCHEN TRUMPF (ANU, CANBERRA)

## A Non-Linear Internal Model Principle for Observers

**7. 7. 2023**, 2pm CEST

**Abstract**

We show that any asymptotic observer for a non-linear repeatable system on a differentiable manifold contains a full internal model of the plant. Examples of repeatable systems are controllable systems and kinematic systems. The result follows from an abstract, set-theoretic internal model principle that generalises the known results for classical LTI systems, behavioural systems, and systems described by injective co-generator signal modules (Oberst's algebraic analysis approach to system theory). At the core of the theory is the use of partially ordered set (poset) theory to generalise the classical LTI system notion of the anti-stabilizable part of a system. Most of the results in this talk were presented at the MTNS 2022 last year

ALFONSO GARMENDIA (UNIVERSITAT POLITÈCNICA DE CATALUNYA)

## PB groupoids and bundle gerbes

**30. 6. 2023**, 2pm CEST

**Abstract**

A Lie 2-group is a groupoid on the category of Lie groups just like Lie groupoids are groupoids in the category of manifolds. To expand this logic, we get PB-groupoids as groupoids in the category of principal bundles, this is, a Lie groupoid with a free and proper action of a Lie 2-group (free and properness implies that the quotient space is still a Lie groupoid). On another context, bundles gerbes are well-studied objects, and are also described as a quotient of a Lie groupoids. In this talk we will introduce more slowly all these objects and prove that see that bundles gerbes sit inside PB-groupoids.

LUCA UMMINGER (JMU WÜRZBURG)

## The star product of Pierre Bieliavsky

**23. 6. 2023**, 2pm CEST

**Abstract**

In this talk we consider the star product of Pierre Bieliavsky $u \star_\hbar v = \tau_\hbar(T_\hbar u \star_{\mathrm{Weyl}} T_\hbar v)$. For this we first consider the space $\tau_\hbar(\mathscr{S}(\mathbb{R}^2)) = \mathcal{E}_\hbar$ on which the star product is defined. We see that $\mathcal{E}_\hbar$ forms a pre-Hilbert space. Furthermore, we show that the star product forms an associative algebra structure on $\mathcal{E}_\hbar$. Moreover, we see that the transvection group acts by algebra automorphism on $(\mathcal{E}_\hbar, \star_\hbar)$ and $(\mathcal{E}_\hbar, \star_\hbar, ^*, \langle\cdot\,,\,\cdot\rangle_{L^2(\mathbb{R}^2)})$ forms a Hilbert algebra. Finally, we give an outlook on how the algebra $(\mathcal{E}_\hbar, \star_\hbar, ^*)$ becomes a C*-algebra.

## Karandeep Singh (KU Leuven)

## Differential graded Lie algebras and stability problems in geometry

**16. 6. 2023**, 2pm CEST

**Abstract**

Given a Poisson structure $\pi$ on a manifold $M$, and a symplectic leaf $L$, the leaf $L$ is stable if every Poisson structure close enough to $\pi$ also has a leaf diffeomorphic to $L$. A sufficient condition for general compact leaves being stable was given by M. Crainic and R. Fernandes. We show that for zero-dimensional leaves, the question of stability is an instance of the following question about differential graded Lie algebras: Given a differential graded Lie algebra $\mathfrak{g}$, a differential graded Lie subalgebra $\mathfrak{h}$, and a Maurer-Cartan element $Q \in \mathfrak{h}$, when is every Maurer-Cartan element of $\mathfrak{g}$ near $Q$ equivalent to an element in $\mathfrak{h}$? In the case that $\mathfrak{h}$ has degreewise finite codimension in $\mathfrak{g}$, we give a sufficient condition for a positive answer to the latter question. We then give some applications of this result to stability of zero-dimensional leaves of other geometric structures.

## Jean-Luc Portner (ETH Zürich)

## Computing the single-valued Knizhnik-Zamolodchikov associator

**9. 6. 2023**, 2pm CEST

**Abstract**

We consider the formula for the single-valued KZ-associator introduced by Rossi and Willwacher. First, we introduce multiple zeta values and Drinfel'd associators. Then, we show that single-valued integration is well suited for calculating the single-valued KZ-associator by example of integrating the wheel graph. On the other hand we see that by comparing coefficients this associator can also be recovered from the KZ and anti-KZ associator. Then we turn to graph complexes and introduce Kontsevichs graph complex as well as show the isomorphism from $H_0(GC)$ to $\mathfrak{grt}_1$ introduced by Willwacher. Finally by combining all the above we can give full descriptions of the single-valued KZ-associator up to weight 13 as well as generators of $\mathfrak{grt}_1$ up to weight 7.

## Alcides Buss (Prodi professor, JMU)

## Landstad duality and Rieffel deformation

**2. 6. 2023**, 2pm CEST

**Abstract**

In this talk we explain the basics of the approach (due to Kasprzak) for Rieffel deformation of actions of locally compact abelian groups on $C^*$-algebras via Landstad duality for those actions and their crossed products. This method can be further extended to coactions of non-abelian groups, and many of the original Rieffel results for actions of $\mathbb{R}^n$ can be generalized to this setting.

Knut Hüper (JMU) and Fátima Silva Leite (University of Coimbra)

## A unified framework for extremal curves on Stiefel manifolds and other homogeneous spaces

**26. 5. 2023**, 2pm CEST


**Abstract**

We present a smooth one-parameter family of submersion (pseudo-)Riemannian metrics on Stiefel manifolds. We discuss their geodesics and the corresponding variational problems together with closed form solutions of the associated Euler-Lagrange equations. We continue with the closed relationship with sub-Riemannian probelems on Lie groups, in particular $\mathrm{SO}_n$. We close with an outlook and recent further work.


Chiara Esposito (Univ. Salerno)

## Reduction in $L_\infty$ terms

**28. 4. 2023**, 2pm CEST


**Abstract**

In this talk we propose a reduction scheme for multivector fields phrased in terms of $L_\infty$-morphisms. First, using geometric properties of the reduced manifolds we perform a Taylor expansion of multivector fields, which allows us to build up a suitable deformation retract of DGLA's. As a second step, we construct a Poisson analogue of the Cartan model for equivariant de Rham cohomology. As a consequence we prove the existence of a curved $L_\infty$ morphism between equivariant multivector fields and multivector fields on the reduced manifolds that coincides with the standard Marsden-Weinstein reduction.


Mahdie Hamdan (Uni Cardiff)

## Non-frustration free ground states of non-abelian quantum double models

**21. 4. 2023**, 2pm CEST


**Abstract**

Quantum spin models are widely studied for their potential use in quantum computing, where they can serve as building blocks for quantum algorithms. Kitaev's quantum double model is of particular interest, as its properties could allow for fault-tolerance quantum computation. In two-dimensional quantum spin systems on the infinite lattice $\mathbb{Z}^2$, this model is known to exhibit a unique frustration-free ground state, although other ground states may exist, and it is theorized that these ground states always correspond to objects in a modular tensor category, deeply connected to the representation theory of the quantum double of an underlying group $G$. Using an operator algebraic approach, I will introduce Kitaevs quantum double model for a finite non-abelian group $G$ on a lattice $\mathbb{Z}^2$ and explore its ground states. I conjecture that these non-frustration free ground states, called anyons, correlate to non-trivial irreducible representations of the quantum double $D(G)$ and that they form a complete set of ground states. This is a joint work with Pieter Naaijkens.

# 2 Winter Term 2022/2023

Martin Bordemann (UHA Mulhouse)

Talk is Cancelled

**10. 2. 2023**, 2pm CET

**Abstract**

The talk is cancelled and will be moved to the summer term.

Gregor Schaumann (JMU Würzburg)

Duality structures and field theory

**3. 2. 2023**, 2pm CET

**Abstract**

In many mathematical areas, there are certain duality operations, for example linear duals of representation spaces, or orientation reversal of manifolds. We discuss Grothendieck-Verdier categories as a framework to describe duals. Often duals are related to Frobenius algebra structures and I try to explain its relation with (conformal) field theory via the representation category of vertex operator algebras. The talk is based on an ongoing project with F. Fuchs, C. Schweigert and S. Wood.

Gandalf Lechner and Ricardo Correa da Silva
(FAU Erlangen-Nürnberg)

Deformations of Araki-Woods algebras

**27. 1. 2023**, 2pm CET (two talks of 45min each)

**Abstract**

Starting from the familiar Weyl algebra on a Bose Fock space over a Hilbert space $\mathcal{H}$, we introduce a family of von Neumann algebras $\mathcal{L}_T(H)$ that can be thought of as deformations of the von Neumann algebra generated by Weyl operators $W(h)$, $h \in H$. Here $H \subset \mathcal{H}$ is a real (standard) subspace of $\mathcal{H}$, and $T$ a "twist" (or deformation), namely a selfadjoint operator on $\mathcal{H} \otimes \mathcal{H}$ satisyfing a positivity condition.

These algebras are called twisted Araki-Woods algebras. They naturally arise in representations of Wick algebras and provide a general framework in which many special cases such as the algebras underlying free Bose fields (on Bose Fock space), free Fermi fields (on Fermi Fock space), integrable QFT models (on S-symmetric Fock space), but also free group factors (on full Fock space) can be discussed in a unified manner.

We will explain the modular theory of these algebras which is closely linked to $T$ being braided and crossing-symmetric, and consider the dependence of $\mathcal{L}_T(H)$ on the twist $T$ (which is quite discontinuous) and the standard subspace $H$. This naturally leads to inclusions $\mathcal{L}_T(K) \subset \mathcal{L}_T(H)$ and applications in QFT.

No deep background in QFT or von Neumann algebras is assumed, only a working knowledge of Hilbert spaces and functional analysis.

Marvin Dippell & Stefan Waldmann (JMU Würzburg)

## An Introduction to `git` in Mathematics

**16. 12. 2022**, 12:30 CET (Note the earlier time!)

**Abstract**

In this little workshop we outline the basic usage of `git` for collaboratively working in mathematics. This becomes interesting when working with other people on common projects (e.g. a paper). However, `git` is already useful for typing a bachelor/master/PhD thesis or lecture notes etc. If you want to participate:

*i.)* Bring your own Laptop

*ii.)* Install Git: Download for Linux and Mac at `https://git-scm.com/` and for Windows at `https://gitforwindows.org/`

*iii.)* You will need access to the gitlab of the University of Würzburg: `https://gitlab2.informatik.uni-wuerzburg.de` If you do not have access yet you can create an account here: `https://user.informatik.uni-wuerzburg.de/getAccount.php` Otherwise, please send us a quick mail then we can create an account for you.

*iv.)* If you have an account, please log in once at `https://gitlab2.informatik.uni-wuerzburg.de` After that, please send us your username, so we can add you to the correct repository.

Marvin Dippell (JMU Würzburg)

## Constraint Reduction in Algebra, Geometry and Deformation Quantization II

**9. 12. 2022**, 2pm CEST/CET

**Abstract**

In this second talk we will study the deformation theory of constraint algebras. For this I will introduce constraint differential graded Lie algebras, their Maurer-Cartan elements and gauge groups. We can then apply these results to a constraint version of the Hochschild complex, which will be seen to control the deformation problem of constraint algebras. I will present some first results towards a constraint Hochschild-Kostant-Rosenberg Theorem, and in particular compute the second constraint Hochschild cohomology in the case of $\mathbb{R}^n$.

Marvin Dippell (JMU Würzburg)

## Constraint Reduction in Algebra, Geometry and Deformation Quantization I

**2. 12. 2022**, 2pm CEST/CET

**Abstract**

Deformation quantization aims at constructing a quantum mechanical analogue to a given classical mechanical system by (formally) deforming the commutative algebra of functions on a given Poisson manifold $(M, \pi)$ into a non-commutative algebra describing the associated quantum system. In this two-piece series I will present a general framework for incorporating the additional information of a coisotropic submanifold $C \subseteq M$ into its algebra of functions and study the deformation theory of these so-called constraint algebras. In the first part, I will focus on the classical

side by introducing constraint manifolds and their vector bundles, as well as constraint algebras and modules as their algebraic analogues. The category of strong constraint projective modules will turn out to be equivalent to the category of constraint vector bundles, giving a constraint version of the Serre-Swan Theorem. If time permits, I will introduce vector fields and differential forms on constraint manifolds, as well as a symbol calculus for constraint multidifferential operators.

Daniele Volpe (University of Trento)

### Paracausal deformations of globally hyperbolic spacetimes and Møller operators in AQFT

**25. 11. 2022**, 2pm CEST/CET

**Abstract**

In recent works with V. Moretti and S. Murro, we constructed a family of isomorphisms relating the solution spaces of normally hyperbolic and Proca operators built out of different globally hyperbolic spacetimes. The isomorphisms extend to the CCR algebras built out of the aforementioned Green hyperbolic operators and preserve the singularity structure of states. Since we were inspired by scattering theory, these isomorphisms are named Møller operators. In order to achieve our goal, we introduced the notion of paracausal relation in the set of globally hyperbolic Lorentzian structures on a fixed differentiable manifold: if two metrics fall in the same equivalence class, then the existence of the Møller operators is guaranteed. In this talk I will summarize our results, focusing on the geometric construction we developed.

Christiaan van de Ven (JMU Würzburg)

### Strict deformation quantization in quantum lattice models

**18. 11. 2022**, 2pm CEST/CET

**Abstract**

Quantization in general refers to the transition from a classical to a corresponding quantum theory. The inverse issue, called the classical limit of quantum theories, is considered a much more difficult problem. A rigorous and natural framework that addresses this problem exists under the name strict (or C*-algebraic) deformation quantization. In this talk, I will first introduce this concept by means of relevant definitions. Next, I will show its connection with the classical limit of quantum theories, starting with a brief summary of the theory in the context of mean-field quantum theories. Finally, I will discuss the results of a recent work on how strict deformation quantization applies to more realistic models described by local interactions for periodic boundary conditions, e.g., the quantum Heisenberg spin chain.

## 3 Summer Term 2022

Babak Dokani Khesroshahi (JMU Würzburg)

### Attempt of an explicit hermitian deformation of the Gutt-star-product on an coadjoint orbit of $\mathfrak{so}(3)$

**15. 7. 2022**, 2pm CEST

**Abstract**

The existence of a hermitian deformation of the Gutt-star-product on a regular coadjoint orbit has been shown for the general case. The proof is constructive and the Lie-Group SO(3) with its Lie-algebra $\mathfrak{so}(3)$ is a promising candidate for an example with an explicit formula for such a deformation. The aim of this representation is to discuss some ideas to achieve such a formula for this example.

Christiaan J.F. van de Ven (JMU Würzburg)

## Quantum Theories and their Classical Limit: a $C^*$-Algebraic Approach

**8. 7. 2022**, 2pm CEST

**Abstract**

Quantization refers to the passage from a classical to a corresponding quantum theory. This notion goes back to the time that the correct formalism of quantum mechanics was beginning to be discovered. The converse problem, called the classical limit of quantum theories, is considered as a much more difficult issue and forms an important question for various areas of modern mathematical physics. As opposed to most traditional methods where one typically aims to obtain quantum mechanics from classical methods, this talk is based on quantization (more precisely, deformation quantization) as the study of the possible correspondence between a given classical theory, defined by a Poisson algebra or a Poisson manifold and possibly a (classical) Hamiltonian, and a given quantum theory, mathematically expressed as a certain algebra of observables or a pure state space. On the basis of this understanding quantization and the classical limit are two sides of the same coin.

In this talk I will present these ideas, first by introducing a natural framework based on the theory of $C^*$-algebraic deformation quantization. Subsequently, I will show how this relates to the classical limit of quantum theories. More precisely, the so-called quantization maps allow to take the limit for $\hbar \to 0$ of a suitable sequence of algebraic states induced by $\hbar$-dependent eigenvectors of several quantum models, in which the sequence typically converges to a probability measure on the pertinent phase space.

In addition, since this $C^*$-algebraic approach allows for both quantum and classical theories, it provides a convenient way to study the theoretical concept of spontaneous symmetry breaking (SSB) as an emergent phenomenon when passing from the quantum realm to the classical world by switching off a semi-classical parameter. This is illustrated with several physical models, e.g. Schrödinger operators and mean-field quantum spin systems.

Stefan Waldmann (JMU Würzburg)

## Between Classical and Quantum: A Student-Friendly Introduction to the Research at ChairX

**1. 7. 2022**, 15:30 CEST

**Abstract**

In this short talk I will give a non-technical overview on the research topics at ChairX. In particular, I will indicate which prerequisites are needed and used.

Markus Schlarb (JMU Würzburg)

## Optimization on Stiefel Manifolds

**1. 7. 2022**, 14:45 CEST

**Abstract**

In this talk, the real (compact) Stiefel manifold $\mathrm{St}_{n,k}$ is considered as an embedded submanifold of the real $(n \times k)$-matrices $\mathbb{R}^{n\times k}$. Explicit formulas for (pseudo-)Riemannian gradients and (pseudo-)Riemannian Hessians are given with respect to a one-parameter family of metrics on $\mathrm{St}_{n,k}$. These formulas allow for the application of Riemannian Newton methods and Riemannian trust-region methods on $\mathrm{St}_{n,k}$.

Martina Flammer (JMU Würzburg)

## Persistent Homology Based Classification of Chaotic Multi-Variate Time Series with Application to EEG Data

**1. 7. 2022**, 2pm CEST

**Abstract**

An application of persistent homology for detection of epileptic events in EEG data is presented. Given point cloud data, persistent homology is a tool from topological data analysis to describe the structure of the underlying space on which the data was sampled by utilizing topological invariants and tracking their behavior on several spatial scales. As a preprocessing step, a novel method called Dynamical Component Analysis is used that reduces the dimension of a multi-variate time series by incorporating information about the dynamics of the system. The results show that our proposed method is appropriate to detect the occurence of petit-mal epileptic seizures in EEG signals.

Michael Heins (JMU Würzburg)

## A slice of Teichmüller theory

**3. 6. 2022**, 2pm CEST

**Abstract**

In this talk, we give a glimpse into one of the numerous approaches to Teichmüller theory, culminating with the Hilbert-Kähler manifold structure of the universal Teichmüller space, as introduced by Teo and Takhtajan. Starting out, we introduce quasiconformal mappings as particular solutions of the Beltrami partial differential equation $\overline{\partial} f = \mu \cdot \partial f$. Imposing a cleverly chosen boundary condition, the corresponding set of solutions turns out to be a somewhat – albeit less than one would like – well behaved group. Passing to a quotient group then yields a definition of the universal Teichmüller space $\mathrm{T}(1)$. Enhancing the quotient topology, we arrive at a complex Banach space atlas. Notably, this atlas may be shrunk to a single chart, the infamous Bers embedding. Finally, one may chop this chart into pieces, each biholomorphic to an immersed Hilbert ball, which induces the Kähler structure by right translation from the tangent space at the group unit. During our voyage, we will meet and promptly skip over various classical results from a diverse range of subfields of analysis.

Gregor Schaumann (JMU Würzburg)

## Quivers and tensor categories

**20. 5. 2022**, 2pm CEST

**Abstract**

Quivers are just finite directed graphs, but by considering their representations one obtains a vast generalization from familiar linear algebra problems. The theory encodes much of the representation theory of finite-dimensional algebras and the representation spaces allow for geometric methods, even applications to symplectic geometry. After introducing some of the main corners of the theory (path algebra, Gabriels theorem, quiver invariants, cohomological Hall algebra), I explain a connection to tensor categories and possible applications.

Matthias Schötz (JMU Würzburg)

## Symmetry reduction of States and a non-commutative Positivstellensatz for $\mathbb{CP}^n$

**13. 5. 2022**, 2pm CEST

**Abstract**

On $\mathbb{CP}^n$ one can construct a (non-formal) star product e.g. by symmetry reduction out of the Wick star product on $\mathbb{C}^{1+n}$. This results in an $\hbar \in \mathbb{C} \setminus \Omega$-dependent associative product $\star_\hbar$ on the space of polynomials (in the sense of real algebraic geometry) on $\mathbb{CP}^n$ which deforms the pointwise product ($\Omega$ is discrete with sole accumulation point 0). For $\hbar \in \mathbb{R} \setminus \Omega$, the pointwise complex conjugation describes a $^*$-involution on this algebra so that one obtains an $\hbar$-dependent family of $^*$-algebras $\mathcal{P}_\hbar(\mathbb{CP}^n)$. One can then try to determine whether or not, or for which values of $\hbar$, these $^*$-algebras $\mathcal{P}_\hbar(\mathbb{CP}^n)$ have non-trivial representations. An essentially equivalent problem is to classify all the algebraic states on $\mathcal{P}_\hbar(\mathbb{CP}^n)$, i.e. all linear functionals $\omega\colon \mathcal{P}_\hbar(\mathbb{CP}^n) \to \mathbb{C}$ that fulfil $\omega(a^* \star_\hbar a) \geq 0$ for all $a \in \mathcal{P}_\hbar(\mathbb{CP}^n)$ and $\omega(1) = 1$. But with the $^*$-algebras $\mathcal{P}_\hbar(\mathbb{CP}^n)$ arising by symmetry reduction out of the well-understood Wick star product, one should rather ask: Under which conditions do algebraic states for the Wick star product on $\mathbb{C}^{1+n}$ descend to algebraic states on $\mathcal{P}_\hbar(\mathbb{CP}^n)$? Can all algebraic states on $\mathcal{P}_\hbar(\mathbb{CP}^n)$ be obtained this way? The latter question leads to a non-commutative Positivstellensatz for $\mathbb{CP}^n$, which, in contrast to its commutative analog, is non-strict.

Markus Schlarb (JMU Würzburg)

## Stiefel Manifolds as Pseudo-Riemannian Submanifolds

**6. 5. 2022**, 15:30 CEST

**Abstract**

In this talk, the real (compact) Stiefel manifold $\mathrm{St}_{n,k}$ is considered from an extrinsic point of view, i.e. as an embedded submanifold of the real $(n \times k)$-matrices $\mathbb{R}^{n\times k}$. A one-parameter family of pseudo-Riemannian metrics on an open $U \subseteq \mathbb{R}^{n\times k}$ is introduced such that $\mathrm{St}_{n,k} \subseteq U$ becomes a pseudo-Riemannian submanifold. This family of metrics on $\mathrm{St}_{n,k}$ coincides with a one-parameter family, called $\alpha$-metrics, which has been recently introduced. Closed form expressions for the orthogonal projections onto tangent spaces can be derived for all metrics in this family. In addition, one obtains explicit formulas for pseudo-Riemannian gradients and pseudo-Riemannian Hessians.

Fátima Silva Leite (University of Coimbra)

### Interpolation on Riemannian Manifolds

**6. 5. 2022**, 2pm CEST

**Abstract**

The problem of finding a smooth curve that interpolates a set of points on a Riemannian manifold, and satisfies some boundary conditions, is particularly important in applied areas such as robotics, computer vision and medical imaging. In this seminar we start with some motivating examples and then will revisit several methods to generate interpolating Riemannian splines. Our main focus will be on a variational approach to generate splines that minimize the intrinsic acceleration, and on a method based on rolling motions that emerged to overcome difficulties in finding explicit solutions for the Euler-Lagrange equation of the previous optimization problem.

Michael Heins (JMU Würzburg)

### Divergence is futile: Entire functions in strict Deformation Quantization

**1. 4. 2022**, 2pm CEST

**Abstract**

Despite the astonishing and far reaching mathematical triumphs of formal Deformation Quantization, there still remains a necessary step to be taken towards a physically applicable theory: Replacing the formal deformation parameter with Planck's constant. In this talk, we discuss one possible approach to *strict* Deformation Quantization, which is due to Waldmann. The principle idea is to show honest convergence of the formal power series with respect to a suitable topology. In practice, one typically proves continuity of the star product on some Poisson subalgebra directly, which solves the convergence problem on its completion. Notably, as a function of $\hbar$, the star product of two fixed elements may exhibit singularities. Both the construction of a suitable subalgebra and the dependence on $\hbar$ naturally lead to the theory of analytic functions. That is to say, complex analysis. We review several observable algebras of strict deformations and argue why they can be understood as incarnations of *entire functions* in their respective contexts.

## 4 Winter Term 2021/2022

Martin Bordemann (UHA Mulhouse)

### Drinfel'd associators for the working algebraist refusing to use more than second year university analysis II

**30. 3. 2022**, 2 am CEST

**Abstract**

The content of the talk is more or less well-known and is focused on an 'elementary' proof of the hexagon and pentagon identities for the famous Drinfel'd associator (V.G.Drinfel'd, 1989/90). The latter is a famous universal formal series in noncommutative monomials of two elements of any complex associative algebra. It applications range from number theory (the coefficients in the series are related to multiple zeta values), to quantization of Lie bialgebras (P.I.Etingof and D.A.Kazhdan, 1996), dequantization of bialgebras and deformation quantization (D.E.Tamarkin, 1999), and quantization of Lie quasibialgebras (B.Enriquez, G.Halbout, 2008). The definition of the associator, and the proof of the hexagon (in any algebra) and pentagon identities (in the Drinfel'd-Kohno algebra) seem to be not quite easy to grasp by the average mortal mathematician, and the original articles and the usual text books on the matter stay somewhat fuzzy about it. The main idea –as in many 'Russian' topics of mathematical physics– is 'monodromy', i.e. to prove identities in certain algebras by representing them as parallel transports over two different composed paths joining the same pair of points in a simply connected domain with respect to some (formally) flat connection (e.g. the one by V.G.Knizhnik, A.B.Zamolodchikov, 1984). The main difficulty is the use of general solutions in terms of 'multi-valued functions' of the linear systems which are only local or global on universal covering spaces (multi-valued functions) which can be inaccessible. We shall just use limits of parallel transports.

My talk presents work in collaboration with Andrea Rivezzi and Thomas Weigel from the University of Milano-Bicocca. In the first talk there will be and introduction about associators and the above-mentioned relations. I shall then review the necessary analysis (hopefully up to second year) which is formal linear differential equations in terms of iterated integrals, then formal connections and parallel transports (where a certain reparametrization invariance plays a crucial role), and some remarks on the norms we shall be using (in fact for each order in the formal parameter we deal with a finite-dimensional complex vector space). At the end of part 1 I hope to introduce the associator as an elementary parallel transport on the interval where the behaviour at the singularities at 0 and 1 can be factorized off while choosing families of paths parametrized by strictly positive real number $\delta$ (which in the end is send to 0).

Part two concerns some properties of the Drinfel'd associator (multiple zeta values) including the hexagon equations which can be derived by parallel transports along simple paths (depending on $\delta$) in the doubly punctured complex plane $\mathbb{C}$. The proof of the pentagon equation has been inspired by a picture in C.Kassel's book 'Quantum Groups', 1995, p.478, which we have translated into concrete paths (depending on $\delta$) in the open subset of $\mathbb{R}^4$ consisting of the points $x_1 < x_2 < x_3 < x_4$. In general, each parallel transport can be factorized into a singular (for $\delta \to 0$) term, a wanted term and a complicated, but harmless term, and in the identities the singular terms cancel, and the complicated harmless terms go 1 (for $\delta \to 0$).

Martin Bordemann (UHA Mulhouse)

## Drinfel'd associators for the working algebraist refusing to use more than second year university analysis I

**29. 3. 2022**, 10am CET

### Abstract

The content of the talk is more or less well-known and is focused on an 'elementary' proof of the hexagon and pentagon identities for the famous Drinfel'd associator (V.G.Drinfel'd, 1989/90). The latter is a famous universal formal series in noncommutative monomials of two elements of any complex associative algebra. It applications range from number theory (the coefficients in the series are related to multiple zeta values), to quantization of Lie bialgebras (P.I.Etingof and D.A.Kazhdan, 1996), dequantization of bialgebras and deformation quantization (D.E.Tamarkin, 1999), and quantization of Lie quasibialgebras (B.Enriquez, G.Halbout, 2008). The definition of the associator, and the proof of the hexagon (in any algebra) and pentagon identities (in the Drinfel'd-Kohno algebra) seem to be not quite easy to grasp by the average mortal mathematician, and the original articles and the usual text books on the matter stay somewhat fuzzy about it. The main idea –as in many 'Russian' topics of mathematical physics– is 'monodromy', i.e. to prove identities in certain algebras by representing them as parallel transports over two different composed paths joining the same pair of points in a simply connected domain with respect to some (formally) flat connection (e.g. the one by V.G.Knizhnik, A.B.Zamolodchikov, 1984). The main difficulty is the use of general solutions in terms of 'multi-valued functions' of the linear systems which are only local or global on universal covering spaces (multi-valued functions) which can be inaccessible. We shall just use limits of parallel transports.

My talk presents work in collaboration with Andrea Rivezzi and Thomas Weigel from the University of Milano-Bicocca. In the first talk there will be and introduction about associators and the above-mentioned relations. I shall then review the necessary analysis (hopefully up to second year) which is formal linear differential equations in terms of iterated integrals, then formal connections and parallel transports (where a certain reparametrization invariance plays a crucial role), and some remarks on the norms we shall be using (in fact for each order in the formal parameter we deal with a finite-dimensional complex vector space). At the end of part 1 I hope to introduce the associator as an elementary parallel transport on the interval where the behaviour at the singularities at 0 and 1 can be factorized off while choosing families of paths parametrized by strictly positive real number $\delta$ (which in the end is send to 0).

Part two concerns some properties of the Drinfel'd associator (multiple zeta values) including the hexagon equations which can be derived by parallel transports along simple paths (depending on $\delta$) in the doubly punctured complex plane $\mathbb{C}$. The proof of the pentagon equation has been inspired by a picture in C.Kassel's book 'Quantum Groups', 1995, p.478, which we have translated into concrete paths (depending on $\delta$) in the open subset of $\mathbb{R}^4$ consisting of the points $x_1 < x_2 < x_3 < x_4$. In general, each parallel transport can be factorized into a singular (for $\delta \to 0$) term, a wanted term and a complicated, but harmless term, and in the identities the singular terms cancel, and the complicated harmless terms go 1 (for $\delta \to 0$).

Chiara Esposito (Univ. Salerno)

## Sullivan models II

**23. 3. 2022**, 2pm CET

**Abstract**

In these seminars we briefly introduce the concept of model of a manifold, in terms of a commutative graded algebra, which is quasi-isomorphic to the de Rham algebra of forms. It is known that the de Rham algebra determines the topological invariants of a manifold but it is somehow hard to manage as it is infinite-dimensional. The rational (real) homotopy theory provides a topological invariant, the so-called model, for compact manifolds which is finite-dimensional.

Chiara Esposito (Univ. Salerno)

## Sullivan models I

**22. 3. 2022**, 10am CET

**Abstract**

In these seminars we briefly introduce the concept of model of a manifold, in terms of a commutative graded algebra, which is quasi-isomorphic to the de Rham algebra of forms. It is known that the de Rham algebra determines the topological invariants of a manifold but it is somehow hard to manage as it is infinite-dimensional. The rational (real) homotopy theory provides a topological invariant, the so-called model, for compact manifolds which is finite-dimensional.

Matthias Frerichs (JMU Würzburg)

## An introduction to the cobordism hypothesis

**18. 2. 2022**, 2pm CET

### Abstract

Topological quantum field theories were first introduced by Atiyah to give an algebraic framework to physical quantum field theories. In his definition a TFT is a functor $Z\colon \mathsf{Bord}_n \to \mathsf{Vect}$, where $\mathsf{Bord}_n$ is the category of $n-1$ dimensional manifolds and bordisms between them. In physical context we would like our field theories to be local in the sense that we are allowed to cut the bordisms into even lower dimensional pieces to compute values of a TFT. This is accounted for by introducing extended TFTs which are now defined on a higher category of $\mathsf{Bord}_n^{\mathrm{fr}}$ of tangentially framed bordisms in which we have a good way to handle embedded submanifolds of codimension 1 to $n-1$. For such a functor to preserve the locality we require, our target category naturally becomes a $(\infty, n)$-category. We will roughly explain these concepts and present the cobordism hypothesis as stated first by Baez and Dolan, which classifies fully extended framed TFTs, stating that manifolds are "freely generated by points". Lastly some specialities in dimension 3 will be discussed which relate to finding a suitable 3 category as target for a TQFT.



Jonas Schnitzer (Univ. Freiburg)


## The strong Homotopy Structure of Phase Space Reduction in Deformation Quantization

**11. 2. 2022**, 2pm CET


### Abstract

A Hamiltonian action on a Poisson manifold induces a Poisson structure on a reduced manifold, given by the Poisson version of the Marsden-Weinstein reduction or equivalently the BRST-method. For the latter there is a version in deformation quantization for *equivariant star products*, i.e. invariant under the action and admitting a quantum momentum map which produces a star product on the reduced manifold.

Fixing a Lie group action on a manifold, one can define a curved Lie algebra whose Maurer-Cartan elements are invariant star products together with quantum momentum maps. Star products on the reduced manifold are Maurer-Cartan elements of the usual DGLA of polydifferential operators. Thus, reduction is just a map between these two sets of Maurer-Cartan elements. In my talk I want to show that one can construct an $L_\infty$-morphism, which on the level of Maurer Cartan elements provides a reduction map.

This is work in progress with Chiara Esposito and Andreas Kraft.



Martina Flammer (JMU Würzburg)


## Persistent Homology and its applications

**4. 2. 2022**, 2pm CET


### Abstract

Persistent Homology (PH) is the main method in topological data analysis, which is a field that can be seen as an intersection of topology, geometry and data analysis. PH arose due to the necessity to describe the structure of spaces on which data was sampled. Using tools from algebraic topology, it provides connectivity information of the underlying space on several spatial scales. In my talk, I will give an overview over the topic of persistent homology, ranging from the original formulation in the early 2000's to its extensions and applications. I will draw special attention to the structure of persistence modules and their generalizations as well as the application of PH to time series analysis.

Thomas Weber (Università del Piemonte Orientale)

## The Hopf Algebra of Christmas Trees and Renormalization of Quantum Field Theory

**17. 12. 2021**, 2pm CET


### Abstract

Christmas trees are one of our favourite aspects of the holidays. Over the centuries, many empirical sciences attempted to understand the phenomenon of those pleasurable plants. While legions of biologists failed, it were as usual the mathematicians who saved the day by uncovering the true nature of Christmas trees. In this talk we consider Christmas trees as mathematical objects and observe that they form a forest, which is endowed with the rich structure of a Hopf algebra, i.e. the quantum analogon of a group. Its multiplication turns out to be commutative, while the coproduct, given by all possible ways to cut down a tree, is not. As an application, we briefly discuss that Christmas trees correspond to divergent integrals in quantum field theory and show how Hopf algebra techniques can be used to renormalize certain divergences. While Christmas trees have been suspected to solve the problem of UV-divergence for quite a while, it were Connes and Kreimer who found this interesting connection.


Bas Janssens (Delft University of Technology)

## Positive energy representations of gauge groups

**10. 12. 2021**, 2pm CET


### Abstract

A projective unitary representation of a gauge group (the group of smooth vertical automorphisms of a principal fibre bundle) is called 'of positive energy' if it admits a compatible representation of the Poincaré group for which the generators of forward timelike translations (the Hamiltonians) have nonnegative spectrum. We show that this (rather natural) condition is remarkably restrictive; it turns out that positive energy representations essentially come from 1-dimensional orbits of the Poincaré group. In particular, if the base manifold is compactified $1 + 1$ dimensional Minkowski space, then positive energy representations can only arise from the conformal boundary. (Joint work with Karl-Hermann Neeb)


Francis Bischoff (University of Oxford)

## Brane quantization of Toric Poisson varieties

**3. 12. 2021**, 2pm CET


### Abstract

The homogeneous coordinate ring of a projective variety may be constructed by geometrically quantizing the multiples of a symplectic form, using the complex structure as a polarization. In this talk, I will explain how a holomorphic Poisson structure allows us to deform the complex polarization into a generalized complex structure, leading to a non-commutative deformation of the homogeneous coordinate ring. The main tool is a conjectural construction of a category of generalized complex branes, which makes use of the A-model of an associated symplectic groupoid. I will explain this in the example of toric Poisson varieties. This is joint work with Marco Gualtieri (arXiv:2108.01658).

## Henrique Bursztyn (IMPA, Rio de Janeiro)

# Revisiting and extending Poisson-Nijenhuis structures

**26. 11. 2021**, 2pm CET


### Abstract

Poisson-Nijenhuis structures arise in various settings, such as the theory of integrable systems, Poisson-Lie theory and quantization. By revisiting this notion from a new viewpoint, I will show how it can be naturally extended to the realm of Dirac structures, with applications to integration results in (holomorphic) Poisson geometry.


## Francesco Fidaleo (JMU Würzburg)

# Modular Spectral Triples and deformed Fredholm modules

**19. 11. 2021**, 2pm CET


### Abstract

Due to possible applications to the attempt to provide a set of equations which unify the four elementary interactions in nature (the grand-unification) and in another, perhaps connected, direction in proving the long-standing, still unsolved, Riemann conjecture about the zeroes of the $\zeta$-function, Connes' non-commutative geometry grew up rapidly in the last decades. Among the main objects introduced (by A. Connes) for handling noncommutative geometry there are the so called spectral triples, encoding most of the properties enjoyed by the (quantum) "manifold" into consideration, and the associated Fredholm modules. On the other hand, the so-called Tomita modular theory is nowadays assuming an increasingly relevant role for several applications in mathematics and in physics. Such a scenario suggests the necessary need to take the modular data into account in the investigation of quantum manifolds. In such a situation, the involved Dirac operators should be suitably deformed (by the use of the modular operator), and should come from non-type $\mathrm{II}_1$ representations. Taking into account such comments, we discuss the preliminary necessary step consisting in the explicit construction of examples of non type $\mathrm{II}_1$ representations and relative spectral triples, called modular. This is done for the noncommutative 2-torus $\mathbb{A}_\alpha$ , provided $\alpha$ is a (special kind of) Liouville number, where the nontrivial modular structure plays a crucial role. For such representations, we briefly discuss the appropriate Fourier analysis, by proving the analogous of many of the classical known theorems in harmonic analysis such as the Riemann-Lebesgue lemma, the Hausdorff-Young theorem, and the $\mathrm{L}^p$-convergence results associated to the Cesaro means (i.e. the Fejer theorem) and the Abel means reproducing the Poisson kernel. We show how those Fourier transforms "diagonalise" appropriately some examples of the Dirac operators associated to the previous mentioned spectral triples. Finally, we provide a definition of a deformed generalisation of "Fredholm module", i.e. a suitably deformed commutator of the "phase" of the involved Dirac operator with elements of a subset (the so-called Lipschitz $*$-algebra or Lipschitz operator system) which, depending on the cases under consideration, is either a dense $*$-algebra or an essential operator system. We also show that all models of modular spectral triples for the noncommutative 2-torus mentioned above enjoy the property to being also a deformed Fredholm module. This definition of deformed Fredholm module is new even in the usual cases associated to a trace, and could provide other, hopefully interesting, applications. The present talk is based on the following papers:



[1] F. Fidaleo and L. Suriano: *Type III representations and modular spectral triples for the noncommutative torus*, J. Funct. Anal. **275** (2018), 1484- 1531.

[2] F. Fidaleo: *Fourier analysis for type III representations of the noncom- mutative torus*, J. Fourier Anal. Appl. **25** (201), 2801-2835.

[3] F. Ciolli and F. Fidaleo: *Type III modular spectral triples and deformed Fredholm modules*, preprint.

Jean Gutt (Institut de Mathématiques de Toulouse)

## On the equivalence of symplectic capacities

**12. 11. 2021**, 2pm CET

**Abstract**

An important problem in symplectic topology is to determine when symplectic embeddings exist, and more generally to classify the symplectic embeddings between two given domains. Modern work on this topic began with the Gromov nonsqueezing theorem, which asserts that the ball symplectically embeds into the cylinder if and only if the radius of the ball is larger than that of the cylinder. Many questions about symplectic embeddings remain open, even for simple examples such as ellipsoids and polydisks. To obtain nontrivial obstructions to the existence of symplectic embeddings, one often uses various symplectic capacities. We shall discuss some questions about capacities, in particular the equality of two type of symplectic capacities. This is joint work with V.Ramos.

Matias del Hoyo (Universidade Federal Fluminense)

## Lie groupoids, Morita equivalences and quantum tori

**5. 11. 2021**, 2pm CET

**Abstract**

Lie groupoids are categorified manifolds, they provide a unified framework for classic geometries, and they can be used to model stacks in differential geometry. Stacks have manifolds, orbifolds, orbit spaces and leaf spaces as examples, and two groupoids present the same stack if they are Morita equivalent. In this talk I will survey the foundations of Lie groupoids, Morita equivalences and differentiable stacks, and present as an application a geometric version of Rieffel's Theorem on quantum tori.

Alejandro Cabrera (Universidade Federal do Rio de Janeiro)

## About quantization and symplectic groupoids

**29. 10. 2021**, 2pm CEST

**Abstract**

In this talk, we will review some recent topics relating quantization of Poisson manifolds and (local) symplectic groupoids. In particular, focusing on the case of a Poisson structure on a coordinate domain, we will explain how analytic Lie-theoretic formulas are related to a ("tree level") part of Kontsevich's star product formula after a suitable Taylor expansion. We will also comment on the relation to the Poisson Sigma Model through a system of PDEs that captures its semiclassical contributions. If time permits, we will also briefly comment on how the integrability into a global symplectic groupoid is reflected on quantizations.

Severin Barmeier (Univ. Köln)

## Strict deformation quantizations of polynomial Poisson structures

**22. 10. 2021**, 2PM CEST

**Abstract**

After Kontsevich's general existence result for formal star products of Poisson manifolds, the convergence of formal star products is an essential but nontrivial next step in the deformation quantization programme. In this talk I will present a combinatorial approach to the quantization of polynomial Poisson structures on $\mathbb{R}^d$ which can be used to obtain star products converging on polynomials. The construction uses the natural $L_\infty$ algebra structure on multi-vector fields obtained by homotopy transfer from the DG Lie algebra structure on the Hochschild complex and Maurer-Cartan elements can be viewed as a systematic way of deforming the commutativity relations of the polynomial algebra. The associated "combinatorial" star product is closely related to the Gutt star product and it admits a graphical description resembling the graphical description of Kontsevich's universal formula. Finally, I will give some examples to illustrate how this star product can be used to obtain strict deformation quantizations of nonlinear Poisson structures by applying a general framework developed by Stefan Waldmann.

This talk will be based on arXiv:2002.10001 joint with Zhengfang Wang and on work in progress joint with Philipp Schmitt.

## 5 Summer Term 2021

Gandalf Lechner (Univ. Cardiff)

### Singular half-sided modular inclusions and the algebra at infinity

**16. 7. 2021**, 2PM CEST

**Abstract**

The concept of a half-sided modular inclusion, i.e. an inclusion of two von Neumann algebras in a particular relative position, is central in the operator-algebraic approach to conformal chiral quantum field theory. Under favourable circumstances, the whole theory on the light ray can be reconstructed from it, including all local algebras and the representation of the Möbius group. An essential requirement is, however, that the inclusion is not singular, i.e. does not have a trivial relative commutant. In this talk I will explain this general setting and then present a technique for producing singular inclusions by a deformation procedure.

Alexander Karabegov (Abilene Christian University)

### Lagrangian fields, Calabi functions, and local symplectic groupoids

**9. 7. 2021**, 4PM CEST (Special time!)

**Abstract**

A Lagrangian field on a symplectic manifold $M$ is a family $\Lambda = \{\Lambda_x | x \in M\}$ of pointed Lagrangian submanifolds of $M$. This notion is a generalization of a real Lagrangian polarization for which each $\Lambda_x$ is the leaf containing $x$. Two Lagrangian fields $\Lambda^s$ and $\Lambda^t$ are called transversal if $\Lambda^s_x$ intersects $\Lambda^t_x$ transversally at $x$ for every $x$. Two transversal Lagrangian fields determine the structure of an almost para-Kähler manifold on $M$. We construct a local symplectic groupoid on a neighborhood of the zero section of $T^*M$ from two transversal Lagrangian fields on $M$. The Lagrangian manifold of $n$-cycles of this groupoid in $(T^*M)^n$ has a generating function which is the $n$-point cyclic Calabi function of a closed (1,1)-form on a neighborhood of the diagonal of $M \times M$ obtained from the symplectic form on $M$.

## Nicolò Drago (Università di Trento)

# KMS functionals and $b$-Poisson manifolds: a guided tour through examples

**18. 6. 2021**, 2 PM CEST

**Abstract**

Given a Poisson manifold $(M, \Pi)$, KMS functionals are certain distributions generalizing the concept of Poisson traces. They provide indirect information on the symplectic foliation associated with $M$. In this talk we will explore the main properties of KMS functionals: in particular we shall provide a fairly complete characterization of such functionals for the case of $b$-Poisson manifolds.

## Francesco Cattafi

# Cartan geometries and multiplicative forms

**11. 6. 2021**, 2 PM CEST

**Abstract**

Cartan connections on principal bundles are often informally referred to as "curved versions" of Maurer-Cartan forms on Lie groups. In modern language, they are defined by special surjective differential form with values in a Lie algebra of the same dimension of the bundle. These objects appear naturally when dealing with geometric structures on manifolds. For instance, the Cartan connection underlying an $n$-dimensional Riemannian manifold can be described as the sum of an $\mathbb{R}^n$-valued form (describing the orthonormal frames) and an $\mathfrak{o}(n)$-valued form (describing the Levi-Civita connection). More generally, any G-structure with the choice of a compatible (Ehresmann) connection defines a Cartan connection, by coupling its tautological form with the connection form.

I will start my talk by reviewing the various concepts mentioned above and their basic properties. I will then introduce an alternative approach to these topics via a class of transitive Lie groupoids equipped with special multiplicative vector-valued differential 1-forms, known as Pfaffian groupoids.

This point of view leads naturally to a more general notion, called a Cartan bundle, which encompasses both G-structures and Cartan geometries as extreme cases. Intuitively, it should be thought of as an "intermediate structure", which interpolate between a "naked" geometric structure, and one endowed with a compatible connection. I will conclude by mentioning further results (joint work in progress with Luca Accornero) and speculations (based on discussions with Andreas Cap).

## Marco Zambon (KU Leuven)

# Deformations of symplectic foliations

**4. 6. 2021**, 2PM CEST

**Abstract**

Symplectic foliations and regular Poisson structures are the same thing. Taking the latter point of view, we exhibit an algebraic structure that governs the deformations of symplectic foliations, i.e. which allows to describe the space of symplectic foliations nearby a given one. Using this, we will address the question of when it is possible to prolong a first order deformation to a smooth path of symplectic foliations. We will be especially interested in the relation to the underlying foliation. This is joint work in progress with Stephane Geudens and Alfonso Tortorella.

Florian Schwarz (JMU Würzburg)

## Pushforward construction for equivariant topological quantum field theories

**21. 5. 2021**, 2PM CEST

**Abstract**

We want to make a $G$-equivariant theory into a $H$-equivariant theory. Here $G$ and $H$ are finite groups and there we use a group-morphism $\lambda\colon G \to H$. A $G$-equivariant topological quantum field theory is a TQFT where the Bordisms have some additional principal bundle structure (a map into the classifying space $BG$).

On the way we will encounter some groupoid gymnastics like principal bundles over a groupoid and weak pullbacks. In the end we will see, that this construction has (at least for some cases) a neat description in terms of defect networks decorated with group elements.

Jim Bryan (University of British Columbia, Vancouver)

## K3 surfaces with symplectic group actions, enumerative geometry, and modular forms

**14. 5. 2021**, 5PM CEST (Later time than usual!)

**Abstract**

The Hilbert scheme parameterizing n points on a K3 surface $X$ is a holomorphic symplectic manifold with rich properties. In the 90s it was discovered that the generating function for the topological Euler characteristics of the Hilbert schemes is related to both modular forms and the enumerative geometry of rational curves on $X$. We show how this beautiful story generalizes to K3 surfaces with a symplectic action of a group $G$. Namely, the Euler characteristics of the "$G$-fixed Hilbert schemes" parametrizing $G$-invariant collections of points on $X$ are related to modular forms of level $|G|$ and the enumerative geometry of rational curves on the orbifold quotient $[X/G]$. The picture also generalizes to refinements of the Euler characteristic such as $\chi_y$ genus and elliptic genus leading to connections with Jacobi forms and Siegel modular forms.

Jan Vysoký (Czech Technical University in Prague)

## Introduction to Graded Manifolds

**07 .05 .2021**, 2PM CEST

**Abstract**

A need for a geometrical theory with integer graded coordinates arose both in geometry (Courant algebroids, Poisson geometry) and physics (AKSZ and BV formalism). Based on the approach of Berezin-Leites and Kostant to supermanifolds, $\mathbb{Z}$-graded manifolds are usually defined as (graded) locally ringed spaces, that is certain sheaves of graded commutative algebras over (second countable Hausdorff) topological spaces, locally isomorphic to a suitable "local model".

This approach works with no major issues for non-negatively (or non-positively) graded manifolds, which is sufficient for most of the applications. However; if one tries to include coordinates of both positive and negative degrees, issues appear on several levels. This was addressed recently by M. Fairon by extending the local model sheaf. Interestingly, this modification creates a new subtle issue on the level of $\mathbb{Z}$-graded linear algebra.

This talk intends to point out the aforementioned issues and to offer the modifications required to obtain a consistent theory of $\mathbb{Z}$-graded manifolds with coordinates of an arbitrary degree.

## Vasily Dolgushev (Temple University, Philadelphia)

## GT-shadows and their action on Grothendieck's child's drawings

**23. 4. 2021**, 2PM CEST


### Abstract

The absolute Galois group of the field of rational numbers and the Grothendieck-Teichmueller group introduced by V. Drinfeld in 1990 are among the most mysterious objects in mathematics. My talk will be devoted to GT-shadows. These tantalizing objects may be thought of as "approximations" to elements of the mysterious Grothendieck-Teichmueller group. They form a groupoid and act on Grothendieck's child's drawings. Currently, the most amazing discovery related to GT-shadows is that the orbits of child's drawings with respect to the action of the absolute Galois group (when they can be computed) and the orbits of child's drawings with respect to the action of GT-shadows coincide! If time permits, I will say a few words about GT-shadows in the Abelian setting. My talk is partially based on the joint paper `https://arxiv.org/abs/2008.00066` with Khanh Q. Le and Aidan A. Lorenz.


## Hessel Bouke Posthuma (University of Amsterdam)

## Deformations of Lie structures and Hochschild cohomology

**16. 4. 2021**, 2PM CEST


### Abstract

Deformations of geometric and Lie structures are typically controlled by a cohomology theory, called deformation cohomology, whereas Hochschild cohomology is associated to deformations of algebras. In this talk I will report on work in progress with Bjarne Kosmeijer relating these two cohomology theories in the setting of Lie groupoids. I will discuss several examples and explain the main result in the light of quantization.


## Chenchang Zhu (Univ. Göttingen)

## Classifying space $BG$ as a (2-shifted) symplectic stack

**9. 4. 2021**, 10AM (!) CEST


### Abstract

It is probably well known to people who know it well that $BG$ carries a sort of symplectic structure, if the Lie algebra of $G$ is quadratic Lie algebra. In this talk, we explore various differential-geometric (1-group, 2-group, double-group) models to realise this (2-shift) symplectic structure in concrete formulas and show the equivalences between them.

In the infinite dimensional models (2-group, double-group), Segal's symplectic form on based loop groups turns out to be additionally multiplicative or almost so. These models are equivalent to a finite dimensional model with Cartan 3-form and Karshon-Weinstein 2-form via Morita Equivalence. All these forms give rise to the first Pontryagin class on $BG$. Moreover, they are related to the original invariant pairing on the Lie algebra through an explicit integration and Van Est procedure. Finally, as you might have guessed, the associated String group $BString(G)$ may be seen as a prequantization of this symplectic structure. From the math-physics point of view, what is behind is the Chern-Simons sigma model. This is a joint work with Miquel Ten Cueca.

## 6 Winter Term 2020/2021

Madeleine Jotz Lean (Univ. Göttingen)

### Transitive double Lie algebroids via core diagrams

**26. 3. 2021**, 2 pm CET

**Abstract**

This talk begins by explaining Brown and Mackenzie's equivalence of locally trivial double groupoids with locally trivial core diagrams (of groupoids). Then it establishes an equivalence between the category of transitive double Lie algebroids and the category of transitive core diagrams (of Lie algebroids). The construction of this equivalence uses the comma double Lie algebroid of a morphism of Lie algebroids, which is introduced as well. The proofs of the results in this talk rely heavily on Gracia-Saz and Mehta's equivalence of decomposed VB-algebroids with super-representations, and they showcase the power of this recent tool in the study of VB-algebroids. Since core diagrams of (integrable) Lie algebroids integrate to core diagrams of Lie groupoids, the equivalences above yields a simple method for integrating transitive double Lie algebroids to transitive double Lie groupoids. This is joint work with Kirill Mackenzie, who sadly passed in 2020.

Simone Gutt (Université Libre de Bruxelles)

### About almost complex structures

**19. 3. 2021**, 2 pm CET

**Abstract**

On any symplectic manifold, there exist compatible positive almost complex structures. Such an almost complex structure is integrable if and only it yields a Kähler structure on the manifold. There are symplectic manifolds which do not admit a Kähler structure. We are interested to know whether properties of a non integrable almost complex structure $J$ on a manifold can select geometric properties of this manifold. To tackle this question, we study distributions naturally defined by the almost complex structure $J$, in particular the one which is spanned at each point by the values of the Nijenhuis tensor associated to $J$. We relate properties of these distributons to the existence of complex substructures on the manifold.

João Nuno Mestre (University of Coimbra)

### Some approaches to the differential geometry of singular spaces

**12. 3. 2021**, 14 Uhr c.t.

**Abstract**

Several objects that appear naturally in differential geometry - the zero set of a smooth function, or the quotient of a manifold by a Lie group action, for example - may not be smooth. But we may still want to study their differential geometry, to the extent possible, in a way that generalizes usual concepts - the zero set of some functions, and the quotient of some group actions are smooth, we want to generalize those.

A few possible approaches are to take inspiration from algebraic geometry and study the object via an appropriately defined algebra, or sheaf, of smooth functions; or maybe to decompose the

object into smaller pieces that are themselves smooth manifolds and fit together nicely; or to describe the object in kind of a "generators and relations" presentation, where the generators and the relations are smooth, and work with the presentation instead. These lead us to the study of differentiable spaces, stratified spaces, and Lie groupoids (which give presentations for differentiable stacks).

In this introductory talk we will see the definitions of these concepts, some examples in which they can be of use, and some classes of singular spaces which are quite well behaved and have good descriptions in all three pictures. I will also try to mention a panoramic view of other approaches to singular spaces, such as diffeological spaces, or noncommutative geometric techniques, and how they relate to the examples presented.

Christiaan van de Ven (University of Trento)

## Asymptotic equivalence of two strict deformation quantizations and applications to the classical limit.

**26. 2. 2021**, 14 Uhr c.t.

**Abstract**

The concept of strict deformation quantization provides a mathematical formalism that describes the transition from a classical theory to a quantum theory in terms of deformations of (commutative) Poisson algebras (representing the classical theory) into non-commutative $C^*$-algebras (characterizing the quantum theory). In this seminar we introduce the definitions, give several examples and show how quantization of the closed unit 3-ball $B^3 \subset \mathbb{R}^3$ is related to quantization of its smooth boundary (i.e. the two-sphere $S^2 \subset \mathbb{R}^3$.) We will moreover give an application regarding the classical limit of a quantum (spin) system and discuss the concept of spontaneous symmetry breaking (SSB) .

Michael Heins (JMU Würzburg)

## The Universal Complexification of a Lie Group

**12. 2. 2021**, 14 Uhr c.t.

**Abstract**

In classical Lie theory, a complexification of a Lie group with Lie algebra $\mathfrak{g}$ is a complex Lie group, whose Lie algebra is given by the complexification $\mathfrak{g}_{\mathbb{C}}$ of $\mathfrak{g}$ in the sense of vector spaces. Both from an analytical and a categorical point of view, this definition turned out to be too naive to be truly useful. Historically, this lead to the refined concept of *universal complexification*, which is based on an analytically desirable universal property. In this talk, we motivate this definition by briefly reviewing the vector space situation. Afterwards, we give a rather geometric construction of the universal complexification of a given Lie group, which was formalized by Hochschild around 1955 and refined by the Bourbaki group in the following decade. Along the way, we review Lie's seminal Theorems and meet the universal covering group. While many properties of the resulting universal complexification align with what we geometrically expect, some notable aspects turn out to differ, which we discuss in detail. Finally, we provide some examples to illustrate the power and limitations of the machinery we have developed.

Marco Benini (Università di Genova)

## Homotopical quantization of linear gauge theories

**5. 2. 2021**, 14 Uhr c.t.

**Abstract**

In a gauge theory, gauge transformations encode a useful higher structure that enables one to perform powerful constructions, e.g. BRST/BV quantization. The efficacy of the BRST/BV approach relies on the flexibility of introducing auxiliary fields, an operation which is formalized by quasi-isomorphisms. This flexibility comes at the price that all constructions must be derived, i.e. invariant under quasi-isomorphisms (as opposed to isomorphisms). Focusing on the prototypical example of linear Yang-Mills theory, I will present a standard model for its derived critical locus and equip the associated complex of linear observables with its canonical shifted Poisson structure (antibracket). I will show how global hyperbolicity of the background Lorentzian manifold entails that this shifted Poisson structure is (homologically) trivial and observe the existence of two distinguished ways to trivialize it. Combining these trivializations leads to a non-trivial unshifted Poisson structure, which I will quantize via canonical commutation relations. This leads to an explicit example of a homotopy algebraic quantum field theory, where the time-slice axiom is encoded weakly by quasi-isomorphisms (as opposed to isomorphisms).

Markus Schlarb (JMU Würzburg)

## Flag Manifolds and Isospectral Matrices

**29. 01. 2021**, 14 Uhr c.t.

**Abstract**

The set of all flags of a fixed signature in $\mathbb{R}^n$ forms a smooth manifold that is called flag manifold. Flag manifolds, which generalize the Grassmann manifolds, can be endowed with the structure of a naturally reductive homogeneous space. Moreover, flag manifolds are diffeomorphic to certain submanifolds of the real symmetric $(n \times n)$-matrices that are given by the orbits of the action of the orthogonal group by similarity transformations. These diffeomorphisms yield the so-called isospectral pictures of flag manifolds. In my talk flag manifolds will be discussed in detail.

Nadja Egner (JMU Würzburg)

## The notion of complete filtered $L_\infty$-algebras

**22. 1. 2021**, 14 Uhr c.t.

**Abstract**

Many problems in deformation theory can be covered by differential graded Lie algebras (DGLA) via the solutions of the Maurer-Cartan (MC) equation modulo gauge action. This is the case for formal Poisson tensors as well as for formal deformations of associative algebras by means of the multivector fields and the Hochschild complex, respectively. However, there are good reasons to consider the broader category of $L_\infty$-algebras instead of DGLA's. In this talk I will present two definitions for $L_\infty$-algebras by coalgebraic techniques leading us to different notions of morphisms.The definition of MC elements can directly be transferred to $L_\infty$-algebras as soon as we guarantee the convergence of series, whereas a generalization of the gauge action requires a bit of work. After establishing these definitions many results on DGLA's can be extended to $L_\infty$-algebras.

## Olaf Müller (Humboldt-Universität zu Berlin)

# New geometrical methods in mathematical relativity

**15. 1. 2021**, 14 Uhr c.t.

### Abstract

This talk presents some new geometric approaches to global behavior of solutions to classical field equations. The results comprise:

- the existence of global solutions for Dirac-Higgs-Yang-Mills Theories (like the standard model) in spacetimes close to Minkowski spacetime in the case of small initial values, via the useful notion of future conformal compactification (joint work with Nicolas Ginoux),
- the existence of maximal Cauchy developments of Dirac-Higgs-Yang-Mills-Einstein theories (e.g. the minimal coupling of the standard model to gravity and its sectors like Einstein-Dirac-Maxwell theory) with the main tool being the Universal Spinor Bundle (joint work with Nikolai Nowaczyk),
- some old and new results about how concentration of energy implies the development of black holes, and the "flatzoomer" method (developped in a joint work with Marc Nardmann) applied in the construction of spacetimes metrics satisfying energy conditions in a given conformal class.

## Martina Flammer

# An Introduction to Persistent Homology

**8. 1. 2021**, 14 Uhr c.t.

### Abstract

Persistent homology is a central topic of the field of topological data analysis (TDA). In TDA one frequently faces the problem that a set of data is given that was sampled on a space but the structure of the space is unknown. Persistent Homology (PH) provides a tool which allows to study the toplogical invariants of this space by building a sequence of simplicial complexes from the data, also known as the persistence complex. By regarding several scales one gets information about connectivity of the space (connected components, holes, voids...), which is can be used for analysis of networks, dynamical systems, protein structures, and much more. In my talk I will give an introduction to the mathematical background, the computation of PH and the state of the art in this field.

## Martin Bordemann (Université de Haute Alsace, Mulhouse)

# Multiplication of differential operators in terms of connections

**18. 12. 2020**, 14 Uhr c.t.

### Abstract

There is a well-known explicit formula for the multiplication of two differential operators in any open set of $\mathbb{R}^n$ in terms of their symbols, by means of the global coordinates $x$ and the additional 'conjugate' coordinates $p$. On a differentiable manifold equipped with a connection $\nabla$ in the tangent bundle any differential operator can be parametrized by a symmetric tensor field paired with symmetrized iterated covariant derivatives (standard ordered or Lichnérowicz prescription,

total symbol calculus). The product of two differential operators can also be written in this form, but the explicit form will contain complicated curvature and torsion terms which in general seem only to be known in terms of the (inverse of the )exponential map of $\nabla$ and parallel transport. The problem is strongly related to the problem of finding explicit formulas for star-products on cotangent bundles of manifolds: these star-products had been treated long time ago by Fedosov, Bordemann/Neumaier/Waldmann, and Bordemann/Neumaier/Pflaum/Waldmann where existence and classification questions had been solved.

In this work we would like to express more explicitly the curvature and torsion terms appearing in the differential operator product. We have chosen the following algebraic approach which seem to work for general commutative rings $K$, $A$ and any morphism $K \longrightarrow A$ as long $K$ and hence $A$ contains the rational numbers.

1. The Lie algebra of all vector fields on a manifold forms a Lie-Rinehart algebra $L$ over the real commutative unital algebra $A$ of all $R$-valued smooth functions on the manifold. The algebra of all differential operators is isomorphic to the so-called universal enveloping algebra $U(L, A)$ of $L$, hence we would like to describe these algebras in general. $A$-linearity (as opposed to just $R$-linearity) can be translated to geometry as 'fibrewise' or 'tensorial'.
2. Connections $\nabla$ à la Koszul can be defined in this framework, as well as their iterations by copying the formulas from differential geometry. It turns out that unsymmetrized iterated covariant derivatives have very pleasant combinatorial properties, so a completely explicit formula can be obtained for symbols being smooth (not necessarily symmetric) tensor fields or elements of $T_A(L)$ in algebraic terms. Here the $A$-linear cocommutative shuffle comultiplication turns out to be a very important piece of structure.
3. The desired enveloping algebra $U(L, A)$ will be a quotient of $T_A(L)$: the two-sided ideal $J(L, A)$ for the only $R$-linear multiplication (which we have to mod out) is also a coideal with respect to the $A$-linear comultiplication which can explicitly be described:
4. In the construction the primitive part of $T_A(L)$ (whose underlying $A$-module is the free $A$-Lie algebra generated by $L$) will become important: it is a Lie-Rinehart algebra over $A$ isomorphic to M.Kapranov's path Lie algebroid (2007). There is a canonical morphism $Z$ of Lie-Rinehart algebras form the primitive part to $L$ whose kernel $H$ carries a representation in $L$ equal to the $A$-Lie algebra of infinitesimal holonomy. There is a recursion equation for $Z$ in terms of curvature and torsion. The coideal $J(L, A)$ is generated $A$-linearly by the kernel $H$.
5. The remaining piece is the projection $T_A(L)$ to $U(L, A) \bmod J(L, A)$ which is a deformation of the usual projection describing the passage to the symmetric algebra $S_A(L)$. It can entirely expressed by rational combinatorics and the map $Z$ which at least partially answers the above problem.

Irina Markina (University of Bergen)

## Evolution of Smooth Shapes and Integrable Systems

**11. 12. 2020**, 14 Uhr c.t.

**Abstract**

We consider a homotopic evolution in the space of smooth shapes starting from the unit circle. Based on the Löwner-Kufarev equation, we give a Hamiltonian formulation of this evolution and provide conservation laws. The symmetries of the evolution are given by the Virasoro algebra. The "positive" Virasoro generators span the holomorphic part of the complexified vector bundle over the space of conformal embeddings of the unit disk into the complex plane and smooth on the boundary. In the covariant formulation, they are conserved along the Hamiltonian flow. The "negative" Virasoro generators can be recovered by an iterative method making use of the canonical Poisson structure. We study an embedding of the Löwner-Kufarev trajectories into the Segal-Wilson Grassmannian, construct the tau-function, and the Baker-Akhiezer function which are related to a class of solutions to the Kadomtsev-Petviashvili hierarchy.

Christoph Schweigert (Hamburg University)

## Topological field theories with boundaries - about tensor networks and Frobenius-Schur indicators

**04.12.2020**, 14 Uhr c.t.


**Abstract**

Topological field theory and, more generally, modular functors, defined on three-manifolds with boundaries has various applications in physics and in mathematics.

In this talk, we explain applications to tensor network states and equivariant Frobenius-Schur indicators.


Marvin Dippell (JMU Würzburg)

## Coisotropic Vector Bundles via Sheaves

**27.11.2020**, 14 Uhr s.t.


**Abstract**

The notion of coisotropic algebra encapsulates the algebraic essence of various reduction procedures used in differential geometry, among others that of reduction of a Poisson manifold by a coisotropic submanifold. The obvious concept of a coisotropic module over a coisotropic algebra has recently been used to identify a category of vector bundle-like structures equivalent to the category of regular projective coisotropic modules, thereby giving a version of the famous Serre-Swan theorem compatible with reduction. One drawback of this approach is that the tangent bundle is not contained in this category. In this talk I will present a different concept of coisotropic vector bundles and give a characterization in terms of locally weakly free coisotropic sheaves of modules.


Andreas Schüssler (JMU Würzburg)

## Algebraic BRST Reduction in Stages

**20. 11. 2020**, 14 Uhr c.t.


**Abstract**

The Marsden Weinstein reduction scheme on a symplectic manifold $(M, \omega)$ in deformation quantization can be modelled by means of a BRST algebra. In case of a product Lie group $G = G_1 \times G_2$ acting on $M$ we will show that BRST algebras allow for a reduction in stages. To do so, we introduce the notion of two compatible BRST structures on the same algebra $\mathscr{A}$ with BRST operators $D_1$ and $D_2$ and show that under certain conditions there exists an isomorphism between $\mathrm{H}^0(\mathrm{H}^0(\mathscr{A}, D_1), D_2)$ and $\mathrm{H}^0(\mathscr{A}, D_1 + D_2)$. Finally, we will show that those conditions are met on a neighbourhood of the constraint surface inside a symplectic manifold.


Pierre Bieliavsky (UC Louvain la Neuve)

## Drinfel'd twists and Rankin-Cohen brackets

**17. 11. 2020**, 15 Uhr c.t.

**Abstract**

Rankin-Cohen brackets are bidifferential operators on modular forms. Certain formal superpositions of these operators have been shown to define associative products on the algebra of modular forms [Eholzer, Cohen-Manin-Zagier, Connes-Moscovici]. I'll present a result which asserts that the associative superpositions of Rankin-Cohen brackets coincide with the formal star products on the plane that are invariant under the linear action of $\mathrm{SL}(2,\mathbb{R})$. Within this context, Eholzer's product equals Moyal's product. I'll give an explicit formula for each of these star products in terms of an integral kernel. This result has the same flavour as El Gradechi's Lie theoretical approach to Rankin-Cohen brackets. All this is a simple application to the special case of the Lie algebra of the affine Lie group "$ax + b$" of a method - which I call the "retract method"- that explicitly constructs the intertwiners between formal Drinfel'd twists sharing a common symmetry.

## Florian Schwarz (JMU Würzburg)

## Classifying spaces of principal bundles

**13. 11. 2020**, 14 Uhr c.t.

**Abstract**

For a topological group $G$, there is an easy classification of all principal $G$-bundles over a space $B$. This classification due to Milnor allows to see every principal bundle over $B$ as the pullback of a universal bundle over a space $BG$. In this talk, the explicit construction of the space $BG$ and the proof for its universal property will be given. In the end we will obtain an equivalence of categories between the mapping groupoid from $B$ to $BG$ and the groupoid of principal bundles over $B$.

## Kasia Rejzner (University of York)

## Algebraic structures in perturbative AQFT

**6. 11. 2020**, 14 Uhr c.t.

**Abstract**

In this talk I will present the main algebraic structures featuring in the perturbative algebraic quantum field theory (pAQFT) framework: the non-commutative star product, the commutative time-ordered product, as well as assorted structures related to the BV quantization. I will focus on the situation, where UV divergences can be neglected (regular observables case). This talk will be partially based on the on-going project with Eli Hawkins.

## Andreas Drotloff (JMU Würzburg)

## Topological quantum field theory and the Reshetikhin-Turaev construction

**30. 10. 2020**, 14 Uhr c.t.

**Abstract**

A key feature that one stumbles across while studying topological quantum field theory (TQFT) is the fact that every $n$-dimensional TQFT gives a representation of the mapping class group of all compact $(n-1)$-dimensional manifold. As for the special case of the 2-dimensional

torus such a representation is also given by a modular tensor category (MTC), the question arises whether 3D TQFT and MTC are connected. This connection is given by the Reshetikhin-Turaev construction which associates a 3D TQFT to every MTC. In this talk, we will give an introduction to the concept of TQFT and discuss how we can obtain invariants of 3-manifolds using an MTC. We then outline how to obtain a full TQFT from these invariants.

Maximilian Stegemeyer (MPI for Mathematics in the Sciences Leipzig)

## Closed Geodesics and String Topology

**23. 10. 2020**, 14 Uhr c.t.

**Abstract**

Given a compact Riemannian manifold without boundary, it is natural to ask how many of the geodesics in the manifold are closed. It is conjectured that on all compact manifolds without boundary and for all Riemannian metrics on them there are infinitely many closed geodesics, however this has not been shown in full generality yet. The standard way to tackle this problem is via the free loop space of the manifold. Morse theory shows that the topology of the free loop space is related to the closed geodesics. In recent years, new algebraic structures on the homology of the free loop space were introduced and it is an interesting question to study the relation of these so-called string topology operations to the closed geodesic problem. This talk gives an introduction to the free loop space and to Morse theory with an outlook towards string topology.

David Roberts

## Mapping stacks of differentiable stacks

**16. 10. 2020**, 14 Uhr c.t.

**Abstract**

The construction of an infinite-dimensional Fréchet manifold of smooth maps out of a compact manifold is well-studied, where the target can be any (fin.dim) smooth manifold. But what if the target is an orbifold? Or more generally a differentiable stack – a stack presented by a Lie groupoid? The 'space', or rather stack, of such maps is the space of fields of a Sigma-model taking values in an orbifold or differentiable stack.

By embedding into the more general diffeological stacks, the mapping stack can be seen, without too much effort, to also be diffeological. But in joint work with Raymond Vozzo, and using a recent technical result from work with Alexander Schmeding, we have shown the mapping stack is presented by a Fréchet Lie groupoid, hence is an infinite-dimensional differentiable stack. We also find, as a side result, that the stack of loops in a (fin.dim.) $U(1)$-bundle gerbe on a manifold $M$ is an infinite-dimensional $LU(1)$-bundle gerbe on $LM$.

Kevin Ruck (JMU Würzburg)

## Hochschild Cohomology and Morita Equivalence II: Picard Groups and their Actions

**9. 10. 2020**, 14 Uhr c.t.

**Abstract**

We now know that there is a well established theorem stating that Morita equivalent algebras have isomorphic Hochschild cohomologies. In the first part of this talk we want to formulate this fact form a more categorical point of view by defining the Picard groupoid and the Hochschild category. In a next step we want to introduce the Picard groups, which are the endomorphism groups for a given object of the Picard groupoid and since the objects are algebras, we can talk about the Picard group of an algebra. In the last part of this talk we will investigate how the Picard group of an algebra acts on the Hochschild cohomology of the same algebra and we will give a closed formula for it in the case of a commutative algebra.

All together

Organizational Meeting

**25. 9. 2020**, 14 Uhr c.t.

**Abstract**

Start-up meeting to organize the seminar for this semester.

## 7 Summer Term 2020

Alexander Spiess (FAU Erlangen-Nürnberg)

Poisson Structures from Poisson-Lie Groupoids and Embedded Graphs

**5. 8. 2020**, 14 Uhr c.t.

**Abstract**

We consider a surface with boundary and define a Poisson structure by labelling an embedded graph with Poisson data. To each vertex we assign a Poisson-Lie groupoid that is equipped with a classical dynamical $r$-matrix. We decorate each edge with a Poisson manifold together with a Poisson action of the groupoids at its start and target vertices. This generalizes Fock and Rosly's Poisson structure, which is used to describe the Poisson structure on the moduli space of flat $G$-bundles on the surface. Fock and Rosly's result requires that the Poisson structure on the moduli space is induced by a classical (non-dynamical) $r$-matrix. We extend this to the case of a classical dynamical $r$-matrix. The Poisson structure on the moduli space is obtained by forming the quotient of our Poisson structure with respect to the groupoid actions at the vertices.

Kian Tadjalli Mehr (JMU Würzburg)

Spherical categories and 3D state-sum models

**24. 7. 2020**, 14 Uhr c.t.

**Abstract**

In this talk we will first introduce the properties of spherical categories and then connect these to geometric properties of three dimensional triangulated manifolds. The goal of this talk is to define a 3D state sum model from a spherical fusion category. Some parts, i.e. triangulations and how they transform into each other, will be similar to my talk from december, but we will go over them again.

Michael Heins (JMU Würzburg)

## Geometric Taylor Formulas on Lie Groups

**10. 7. 2020**, 14 Uhr c.t.

**Abstract**

Classical Taylor expansions are ubiquitous in the mathematical modelling of systems in the natural sciences, which can be described by vector spaces. While on a manifold this still works within an analytic chart, it is not clear how to extend the formulas beyond this local situation. In this talk, we provide two answers to this question for analytic functions on a Lie group. To this end, we first reformulate the local Taylor formula in a geometric fashion. In a second step we globalize the local result to incorporate the geometry of the Lie group. As all methods are elementary differential geometry, we will provide complete proofs. Finally, we draw a connection to the algebras of possibly trivial functions introduced in an earlier talk.

Thomas Weber (Universitá degli Studi del Piemonte Orientale Amedeo Avogadro)

## The Braided Cartan Calculus and Braided Commutative Geometry

**3. 7. 2020**, 14 Uhr c.t.

**Abstract**

A manifold with Lie algebra symmetry admits a Drinfel'd twist deformation quantization if there is a normalized 2-cocycle on the corresponding universal enveloping algebra. Furthermore, every tensor field on the manifold can be quantized in a compatible way, using the same twist. The procedure is functorial, leading to a noncommuative Cartan calculus. In this talk we generalize this well-known construction to braided commutative algebras with triangular Hopf algebra symmetries. The Drinfel'd functor corresponds the equivalence classes in this setting. After discussing the Cartan relations we introduce equivariant metrics and covariant derivatives in braided commutative geometry. We prove existence and uniqueness of a braided Levi-Civita covariant derivative for any non-degenerate metric and show how the Drinfel'd functor intertwines the construction.

Kevin Ruck (JMU Würzburg)

## Hochschild Cohomology and Morita Equivalence

**26. 6. 2020**, 14 Uhr c.t.

**Abstract**

Morita equivalence can be considered as an extension of the notion "isomorphic algebras" from the commutative case to the noncommutative case. In fact, it is known that Morita equivalent algebras share a long list of common algebraic properties, the so called Morita invariants. In this talk we want to focus on a very specific Morita invariant, the Hochschild Cohomology. We will start by giving a comprehensive introduction to the theory of Hochschild Cohomology and its applications in the study of formal deformations of algebras. After defining Morita equivalence in a very hands on fashion, we will then present the well established theorem, which shows that the Hochschild Cohomologies of Morita equivalent algebras are in fact isomorphic, hence adding it to the list of Morita invariants as claimed before.

Andreas Drotloff (JMU Würzburg)

## Modular tensor categories and $\mathrm{SL}_2(\mathbb{Z})$

**19. 6. 2020**, 14 Uhr c.t.

**Abstract**

The modular group $\mathrm{SL}_2(\mathbb{Z})$ has a well known connection to surface theory as the mapping class group of the torus. But it also makes an appearence in the study of tensor categories, more precisely in the special case of modular tensor categories (MTC). This may seem surprising at first, but it can be explained by the fact that Modular tensor categories provide a way to classify extended 3-dimensional topological field theories via a construction given by Reshetikhin and Turaev. In this talk, we will discuss monoidal categories and introduce various additional structures like duality and braiding. These will enable us to state the definition of an MTC and examine how every MTC gives a projective representation of $\mathrm{SL}_2(\mathbb{Z})$.

Felix Menke (JMU Würzburg)

## A Coisotropic Serre-Swan Theorem

**12. 6. 2020**, 14 Uhr c.t.

**Abstract**

The procedure of coisotropic reduction known from Poisson geometry can be formulated algebraically using the concept of coisotropic algebras. In this talk, an introduction to this approach is given. We will proceed with studying the corresponding notion of coisotropic modules over coisotropic algebras and focus on *projective* coisotropic modules. Ultimately, we will find a Serre-Swan-like geometric description of the projective coisotropic modules over the coisotropic algebra occuring in Poisson coisotropic reduction.

Michael Heins (JMU Würzburg)

## A Possibly Trivial Strict Deformation Quantization

**5. 6. 2020**, 14 Uhr c.t.

**Abstract**

Lie groups are ubiquitous in mathematical physics. For example, the mechanics of a rigid body can be described by the cotangent bundle of the product of Lie groups $\mathbb{R}^n \times \mathrm{SO}(n)$. As Lie groups admit for a global frame by left translation, many constructions involved in their quantization simplify compared to general cotangent bundles. In this talk, we discuss a new example of a strict deformation quantization of a suitably chosen and possibly trivial Poisson subalgebra of the polynomials $\mathrm{Pol}(T^*G) = \mathscr{C}^\infty(G) \otimes \mathrm{Pol}(\mathfrak{g}^*)$. This factorization allows us to build on a strict deformation of the Gutt star product on Lie algebras worked out by Stapor in 2016. Along the way, we review the first factor in detail and key properties of the second one. Furthermore we encounter projective tensor products, the half-commutator covariant derivative and the associated standard ordering. Their study ultimately results in a direct continuity estimate for the corresponding star product. Finally, we discuss some of our current ongoing attempts at ensuring the subalgebra we use is non-trivial, illustrated by examples from the flat case.

## Gregor Schaumann (JMU Würzburg)

# Morita theory for tensor categories

**29. 5. 2020**, 14 Uhr c.t.

**Abstract**

Tensor categories can be regarded as a categorification of $^*$-algebras. We discuss several aspects of this correspondence regarding balanced tensor products of modules, algebra valued inner products and Picard groupoids. Additionally new features arise for tensor categories that are not present for $^*$-algebras, where we discuss pivotal structures, unimodularity, etc.

## Martina Flammer (JMU Würzburg)

# Motion and Image Reconstruction in Medical Applications

**22. 5. 2020**, 14 Uhr c.t.

**Abstract**

Magnetic Resonance Imaging (MRI) of the lungs proves to be difficult due to the respiratory motion appearing during the imaging process. To overcome this obstacle, a variational model that incorporates motion information in the image reconstruction is used. The motion information is obtained by determining the optical flow in the image sequence. For the image reconstruction, a radial sampling scheme is assumed and therefore the Radon transformation is used as forward operator. Additionally, the variational approach copes with strongly undersampled MRI data. In comparison to static reconstruciton methods, the proposed algorithm gives better results, which was tested on a real MRI data set provided by the University Hospital in Würzburg.

## Nicolò Drago (JMU Würzburg)

# Ricci flow and algebraic quantum field theory

**15. 5. 2020**, 14 Uhr c.t.

**Abstract**

The Ricci flow has been introduced in the mathematical literature in the early eighties by Hamilton. It also appeared independently in the context of quantum field theory thanks to the early works of Friedan within the analysis of nonlinear Sigma models (NLsM). Therein, the Ricci flow has been heuristically derived from the anomalous scaling behaviour of the Lagrangian underlying the theory. In this talk we present a rigorous derivation of the Ricci flow on firm mathematical grounds. For that, we will discuss the algebraic approach to NLsMs and the concept of renormalization group.

## Andreas Schüssler (JMU Würzburg)

# Involutions in Reduction

**8. 5. 2020**, 14 Uhr c.t.

**Abstract**

In the quantum version of the Marsden-Weinstein reduction procedure one constructs a star product on the reduced manifold $M_{\mathrm{red}}$ for which the complex conjugation happens to be a $^*$-involution. To include an involution into the reduction procedure itself two different ways will be discussed in this talk. One uses techniques of BRST-algebras with involutions yielding the complex conjugation on the reduced function space if the acting Lie group is compact. The other approach uses a positive funcional to induce an involution on $\mathscr{C}^\infty(M_{\mathrm{red}})[[\lambda]]$ which in general is not the complex conjugation.

## All together

## Planning of the DQ Seminar

**24. 4. 2020**, 14 Uhr c.t.

**Abstract**

In this first meeting we will try the new techniques of online seminars. Access to the seminar room is on request, please write us an email. Moreover, we make a first schedule for the talks this semester, volunteer welcome!

# 8 Winter Term 2019/2020

Michael Heins (JMU Würzburg)

## Strict deformation quantization: from convergence to continuity

**7. 2. 2020**, 14 Uhr c.t.

**Abstract**

With the flourish of formal deformation quantization a lot of formal star products have been constructed. In recent years examples were found, in which the star product can be understood as convergent in a suitable topology However, these are few and far between with no unified theory in sight. We discuss the easiest non trivial example of such a strict deformation in the form of the Weyl-Moyal star product on $T^*\mathbb{R}^n$. Furthermore we argue why asking for continuity is ultimately the better question and outline a general strategy to approach these problems with. On the way we will discuss recent results by Waldmann, Stapor and Winklmaier-Peran.

Eva Horlebein (JMU Würzburg)

## Spin structures, Stiefel-Whitney classes and Morita equivalence

**31. 1. 2020**, 14 Uhr c.t.

**Abstract**

In this talk a way to define the first and second Stiefel-Whitney class of a vector bundle in terms of its transition functions shall be given and the connection to orientability and spin structures will be shown. In the end shall be discussed how this and the Whitney sum formula of Stiefel-Whitney-classes may be used to obtain a Morita equivalence result about section bundles of real Clifford algebras.

Felix Menke (JMU Würzburg)

## Vector Bundles in Phase Space Reduction

**24. 1. 2020**, 14 Uhr c.t.

**Abstract**

For example in coisotropic reduction one obtains a reduced phase space $M/\mathscr{F}$ by considering the leaf space of a suitable foliation $\mathscr{F}$ of a manifold $M$. In this talk we will describe how the vector bundles over such reduced phase spaces look like in terms of vector bundles over the manifold $M$: Here we have to construct a vector bundle over $M/\mathscr{F}$ having given a vector bundle over $M$. For this, we develop a notion of parallel transport in the direction of leaves of a foliation using only partially defined covariant derivatives. In a next step, we define a suitable category of vector bundles over $M$ which turns out to be equivalent to the category of vector bundles over $M/\mathscr{F}$.

Eske Ewert (Universität Göttingen)

## Tangent Groupoid and Pseudo-Differential Operators

**17.01.2020**, 14 Uhr c.t.

**Abstract**

For a manifold $M$, the C*-algebra of Connes' tangent groupoid is a continuous field of C*-algebras. It deforms the compact operators on $L^2(M)$ into the algebra of continuous functions on the cotangent bundle of $M$. I will explain how pseudo-differential operators of order zero on $M$ can be obtained as generalized fixed points of a scaling action of $\mathbb{R}_{>0}$ on the tangent groupoid.

Moreover, I will give an outlook how the same construction yields a pseudo-differential calculus if $M$ is replaced by a homogeneous Lie group. Here, the algebra of principal symbols turns out to be non-commutative, but our approach allows to compute its K-theory.

Andreas Kraft (University of Salerno)

## Equivariant Formality and Ideas for Reduction

**10.01.2020**, 14 Uhr c.t.

**Abstract**

In the context of deformation quantization equivariant star products are the quantum analogue of Hamiltonian group actions with momentum maps. We recall their classification by an equivariant formality and sketch a reduction procedure on the classical side of equivariant polyvector fields. If time permits, we will give ideas towards a similar reduction at the level of polydifferential operators and how this could help to understand the question of commutativity of reduction and quantization.

All together

## Xmas@ChairX

**20. 12. 2019**, 14 Uhr c.t.

**Abstract**

Kian Tadjalli Mehr

## An algebraic description of the 2d state-sum model II

**13.12.2019**, 14 Uhr c.t.

**Abstract**

We begin this talk by defining triangulation of two dimensional surfaces and introducing piecewise-linear manifolds. This concept will be used to construct a state-sum model for closed, compact 2d manifolds. Then we will show how demanding independence of the model under change of triangulation translates into algebraic properties of the state-sum variables. Finally, there will be a short introduction to cobordisms, to understand how to describe the state-sum in the non-closed case of manifolds with boundary.

## Kian Tadjalli Mehr

# An algebraic description of the 2d state-sum model

**06.12.2019**, 14 Uhr c.t.


**Abstract**

We begin this talk by defining triangulation of two dimensional surfaces and introducing piecewise-linear manifolds. This concept will be used to construct a state-sum model for closed, compact 2d manifolds. Then we will show how demanding independence of the model under change of triangulation translates into algebraic properties of the state-sum variables. Finally, there will be a short introduction to cobordisms, to understand how to describe the state-sum in the non-closed case of manifolds with boundary.


## Ricci Flow from Euclidean Algebraic Quantum Field Theory

# Paolo Rinaldi

**29. 11. 2019**, 14 Uhr c.t.


**Abstract**

The perturbative approach to nonlinear Sigma models and the associated renormalization group flow are discussed within the framework of Euclidean algebraic quantum field theory and of the principle of general local covariance. In particular we show in an Euclidean setting how to define Wick ordered powers of the underlying quantum fields and we classify the freedom in such procedure by extending to this setting a recent construction of Khavkine, Melati, and Moretti for vector valued free fields. As a by-product of such classification, we provide a mathematically rigorous proof that, at first order in perturbation theory, the renormalization group flow of the nonlinear Sigma models is the Ricci flow.


## Maximilian Stegemeyer

# How to find the right path for the SPD (matrices) and a unique principal connection on the Stiefel manifold.

**22. 11. 2019**, 14 Uhr c.t.


**Abstract**

This talk consists of two parts. First, we apply results on endpoint geodesics to the symmetric positive definite matrices. This is a non-compact symmetric space and we use an embedding that is not the standard embedding. In the second part, a result on principal connections on the Stiefel manifold is presented. The Stiefel manifold is a principal fiber bundle over the Grassmannian and there is a natural transitive left action of $SO_n$ that is compatible with the principal action. Therefore we study principal fiber bundles with an additional compatible transitive left action. It turns out that there is a unique $SO_n$-invariant principal connection on the Stiefel manifold.


## Nicolò Drago (JMU Würzburg)

# A friendly chat on classical KMS states

**15. 11. 2019**, 14 Uhr c.t.

**Abstract**

This is an introductory talk on Kubo Martin Schwinger (KMS) classical states. Given a Poisson manifold $(M, \pi)$, KMS states are linear positive functionals $\varphi\colon C^\infty_{\mathrm{c}}(M) \to \mathbb{C}$ generalizing the notion of Poisson traces. They describe thermal equilibrium at a fixed temperature for the classical system described by $(M, \pi)$. In this talk we will define KMS states and discuss their basic properties as well as their connection with the underlying Poisson structure.

Markus Schlarb

## Smooth Generalized Subbundles and Integrability of Smooth Distributions with Singularities II

**8. 11. .2019**, 14 Uhr c.t.

**Abstract**

Smooth not necessarily regular distributions and their integrability play a fundamental role in some areas of differential geometry.

In the first talk smooth generalized subbundles of vector bundles of which smooth distributions are a special case will be introduced. An example is the image of a vector bundle morphism. Regarding the very definition of a generalized subbundle, it might be surprising that in fact every generalized subbundle is of this form. I will show that for an arbitrary smooth generalized subbundle there exists a suitable vector bundle morphism such that its image is the given generalized subbundle.

The second talk will deal with the integrability of smooth not necessarily regular distributions. In the singular case the well-known Frobenius theorem fails in general. However, there are characterizations of smooth integrable distributions generalizing this theorem which are valid in the singular case, too. They are usually known as Stefan-Sussmann theorem. Some of them will be discussed after a brief introduction to the notion of integrable distributions.

Markus Schlarb

## Smooth Generalized Subbundles and Integrability of Smooth Distributions with Singularities I

**25. 10. .2019**, 14 Uhr c.t.

**Abstract**

Smooth not necessarily regular distributions and their integrability play a fundamental role in some areas of differential geometry.

In the first talk smooth generalized subbundles of vector bundles of which smooth distributions are a special case will be introduced. An example is the image of a vector bundle morphism. Regarding the very definition of a generalized subbundle, it might be surprising that in fact every generalized subbundle is of this form. I will show that for an arbitrary smooth generalized subbundle there exists a suitable vector bundle morphism such that its image is the given generalized subbundle.

The second talk will deal with the integrability of smooth not necessarily regular distributions. In the singular case the well-known Frobenius theorem fails in general. However, there are characterizations of smooth integrable distributions generalizing this theorem which are valid in the singular case, too. They are usually known as Stefan-Sussmann theorem. Some of them will be discussed after a brief introduction to the notion of integrable distributions.

Marisa Schult

# Kähler reduction for arbitrary regular values and the shifting trick

**18. 10. 2019**, 14 Uhr c.t.

**Abstract**

In this talk, I will shortly revise the basic notation for the Marsden-Weinstein Reduction Theorem for symplectic manifolds before expanding the underlying theory to a new type of manifold. I will also point out important constructions used in this process and finally re-introduce the resulting reduction theory for Kähler manifolds. Even though this reduction procedure is known for both cases it is as yet not clear if a related construction from symplectic geometry, namely the shifting trick, also works in the Kähler situation. I will explain the shifting trick construction in both settings. However, it turns out that there is a difference between the two cases. Even though there exists a symplectomorphism between the resulting quotient manifolds it can be shown that this map is not isomorphic.

# 9 Summer Term 2019

Maximilian Stegemeyer (JMU Würzburg)

## Endpoint geodesics on symmetric spaces

**19. 7. 2019**, 14 Uhr c.t.

**Abstract**

Symmetric spaces are among the most prominent examples of Riemannian manifolds since they exhibit many beautiful features. They can be viewed from different points of view, one of them a Lie theoretic one, which describes them as special homogeneous spaces. With this approach it is possible to find a formalism for isometric embeddings of certain compact symmetric spaces. Geodesics on symmetric spaces are well understood but it is often an interesting problem to consider the geodesic equation as a boundary value problem. With the isometric embedding mentioned above, it is possible to solve this problem. The talk will give a brief introduction to symmetric spaces and then present results on endpoint geodesics.

Maurice de Gosson (Prodi Professor, Univ. Wien)

## A Geometric Characterization of the Separability of Gaussian Quantum States

**12. 7. 2019**, 12.30 Uhr s.t.

**Abstract**

We begin by discussing the notion of partial tracing from the point of view of the Wigner formalism. We thereafter study the orthogonal projections ("shadows") of the covariance ellipsoids of Gaussian mixed states on the partial phases spaces; this allows us to relate these results to deep results in symplectic topology.

Tobias Schmude (JMU Würzburg)

## Idempotent Completion and Formal Theory of Monads

**5. 7. 2019**, 14 Uhr c.t.

**Abstract**

Nils Carqueville (Uni Wien)

## Defect TQFTs and orbifolds

**04.07.2019**, 14 Uhr c.t.

**Abstract**

A general framework will be discussed which unifies group equivariantisations and state sum models, in the context of topological quantum field theory (TQFT) in arbitrary dimension. After a review of the 2-dimensional case, I will outline general aspects of the construction and and discuss examples in 3 dimensions, including Turaev- Viro theory as and orbifold, as well as surface defects in quantised Chern-Simons theory. (Based on joint work with I. Runkel and G. Schaumann.)

David Kern (JMU Würzburg)

## From a Lie Algebroid Morphism to a Star Product Morphism

**28. 6. 2019**, 14 Uhr c.t.

### Abstract

This talk start with a repetition of Lie algebroids and their comorphisms mentioned in the first talk which will be completed by introducing Lie algebroid morphisms. Nevertheless, there is still another kind of morphism for Lie algebroids which is called Lie algebroid morphism and is equivalent to a morphism between differential graded algebras. Furthermore, Lie algebroid derivatives will be introduced in this talk. They are a generalization of covariant derivatives of the tangent bundle of the underlying base manifold and there is a way to related different Lie algebroid derivatives for different base manifolds. All this is nessessary to understand more about the Fedosov construction for Lie algebroids that are related by a Lie algebroid morphism. The aim of this talk is to give a morphism between star product algebras coming from a Lie algebroid morphism.

Marvin Dippell (JMU Würzburg)

## Vector Bundles and their Sections: An Introduction to Fibred Categories

**21.06.2019**, 14 Uhr c.t.

### Abstract

The well-known Serre-Swan theorem reveals a deep connection between vector bundles over smooth manifolds and finitely generated projective modules, and thus between geometry and algebra. At first glance this theorem only holds for vector bundle morphisms over the identity, since one can not simply pull back vector fields. But one can pull back one-forms! So instead of taking sections of vector bundles directly we are led to take sections of the dual bundle. The conceptual problem is now that taking dual bundles seems not to be functorial. A framework in which this issue can be tackeld and, even better, solved is that of fibred categories. I will give an introduction to fibred cateogires, cleavages and fibrewise duals based on examples from differential geometry. If time permits connections to other topics from differential geometry like Lie algebroids and covariant derivatives will be sketched.

Eva Horlebein (JMU Würzburg)

## Clifford algebras and spin structures

**14. 6. 2019**, 14 Uhr c.t.

### Abstract

Clifford algebras and spin structures play an important role in many fields of mathematics and physics. To allow to work with these constructions, the beginning of the talk deals with defining what Clifford algebras, spin groups and spin structures are and with giving some basic properties. This shall enable to discuss a theorem by Plymen that states an equivalence between the existence of a complex spin structure on a vector bundle and the existence of an isomorphism between its complex Clifford bundle and a suitable endomorphism bundle.

S. Ivan Trapasso (Politecnico di Torino)

## Pointwise convergence of the integral kernels of Feynman path integrals

**13. 6. 2019**, 12 Uhr c.t.

**Abstract**

The Feynman path integral formulation of quantum mechanics is universally recognized as a milestone of modern theoretical physics. Roughly speaking, the core principle of this picture provides that the integral kernel of the time-evolution operator shall be expressed as a "sum over all possible histories of the system". This phrase entails a sort of integral on the infinite-dimensional space of suitable paths, to be interpreted in some sense as the limit of finite-dimensional short-time approximation operators. In spite of the suggestive heuristic arguments and the success as a practical tool for performing computations [4], the quest for a rigorous derivation of the Feynman path integrals is far from over. This is evidenced by the wide variety of attempts to give mathematical meaning to this framework, mainly with the equipment of functional, harmonic and stochastic analysis [1, 2].

Notwithstanding the several outcomes concerning the convergence in suitable operator topologies, the original Feynman's idea underlay the much more difficult and widely open problem of the pointwise convergence of the integral kernels of the approximation operators [3]. We address this problem and significantly benefit from concepts and techniques arising in the context of time-frequency analysis, which have been fruitfully applied to the study of path integrals only in recent times [5, 6, 7].

We consider path integrals in the Trotter-type form for the Schrödinger equation, where the Hamiltonian is the Weyl quantization of a real-valued quadratic form perturbed by a bounded non-smooth potential [8]. In a nutshell, we rephrase the problem in terms of pseudodifferential operators and then exploit the rich structure enjoyed by certain function spaces of a marked harmonic analysis flavour, defined in terms of the decay of the Fourier transform, namely the modulation spaces $M^\infty_s(\mathbb{R}^{2d})$ (with $s > 2d$) and $M^{\infty,1}(\mathbb{R}^{2d})$. In particular, they are Banach algebras for both pointwise multiplication and product of symbols for the Weyl calculus.

*Joint work with Fabio Nicola.*

1. S. Albeverio, R. Høegh-Krohn, and S. Mazzucchi: *Mathematical theory of Feynman path integrals. An Introduction.* Lecture Notes in Mathematics 523. Springer-Verlag, Berlin, 2008.
2. S. Albeverio, and S. Mazzucchi: Path integral: mathematical aspects. Scholarpedia, **6**(1):8832, 2011.
3. R. Feynman and A.R. Hibbs: *Quantum Mechanics and Path Integrals.* Emended Edition. Dover Publications, Mineola, 2005.
4. C. Grosche and F. Steiner: *Handbook of Feynman path integrals.* Springer, Berlin, 1998.
5. F. Nicola: Convergence in $L^p$ for Feynman path integrals. *Adv. Math.* **294** (2016), 384–409.
6. F. Nicola: On the time slicing approximation of Feynman path integrals for non-smooth potentials. *J. Anal. Math.* **137**(2) (2019), 529–558.
7. F. Nicola, and S. I. Trapasso: Approximation of Feynman path integrals with non-smooth potentials. arXiv preprint, arXiv:1812.07487, 2018.
8. F. Nicola, and S. I. Trapasso: On the pointwise convergence of the integral kernels in the Feynman-Trotter formula. arXiv preprint, arXiv:1904.12531, 2019.

## Simone Gutt (Université Libre de Bruxelles)

# Natural almost complex structures on twistor spaces

**7. 6. 2019**, 14 Uhr c.t.

**Abstract**

The twistor space $J(M)$ of a $2n$-dimensional Riemannian or symplectic manifold $M$ is the bundle consisting of all compatible almost structures on $M$. It is endowed with two natural almost complex structures $J^{\pm}$. The structure $J^{+}$ was used by Atiyah on 4-dimensional oriented riemannian manifolds; in particular conditions were given for $J^{+}$ to be integrable. The structure $J^{-}$ was introduced by Eells and Salamon as a first example of natural non integrable almost complex structure. We shall present nice tools introduced by O'Brian and Rawnsley to study those spaces, and show cases where $J^{-}$ is maximally non integrable.

## Tom Weber (Univ. Hamburg)

# Modified trace and Renormalised Hennings Invariant

**20. 5. 2019**, 14 Uhr c.t.

**Abstract**

A modified trace on an ideal of a pivotal $k$-category is a linear map satisfying cyclicity and partial trace property. The modified trace on the representation category $C$ of a finite-dimensional factorisable Hopf algebra $H$ gives rise to an invariant of 3-manifolds with embedded $C$-coloured graphs. If $H$ is non-semisimple the invariant can distinguish certain Lens spaces.

## Mahdi Hamdan (JMU Würzburg)

# Morita Theory for Locally Convex Algebras

**17. 5. 2019**, 14 Uhr c.t.

**Abstract**

Algebraic objects are often examined, by considering their categories of representations. So the question arises, whether two algebras have equivalent categories of representations, rather than them being isomorphic. We call two algebraic objects Morita equivalent, if their categories of representations are. To that extend, Morita theory for purely algebraic objects, like rings or algebras, are already well understood. In practice however, one often encounters additional topological properties. In my talk, we will consider the category LCAlg of non-unital locally convex algebras who posess an approximation of unity. As it is well known from the theory of unital rings, one can introduce a notion of Morita equivalence by using the notion of a conveniently defined bicategory. By introducing a well behaved tensor product on LCAlg one gets a monoidal category, from which one can construct a bicategory BiCat, whose objects being algebras, 1-morphisms being bimodules over these algebras and 2-morphisms being morphisms of said bimodules. If the bicategory is chosen correctly, two algebras will be Morita equivalent, if and only if they are equivalent as objects in BiCat. In the case of muliplicative Fréchet-algebras with approximate unity, it will turn out that the projective tensor product is the correct choice.

# 10 Winter Term 2018/2019

Marisa Schult (JMU Würzburg)

## From Symplectic to Kähler Geometry –Quotients–

**8. 2. 2019**, 14 Uhr c.t.

**Abstract**

The Marsden-Weinstein Reduction Theorem is a well understood way of equipping the quotient space of a symplectic manifold by a (real) Lie group action once again with a symplectic structure. One might now ask what additional properties of such an action would suffice to ensure that a similar construction is possible in the Kähler case. To shed some light on a viable answer we first recall the notions of Lie group actions and momentum maps. The main part focuses on the construction of a compatible Kähler structure on the quotient of a Kähler manifold by a suitable (real) Lie group action.

Christoph Bender (JMU Würzburg)

## Riemann surfaces and proper holomorphic embeddings – a summary

**1. 2. 2019**, 14 Uhr c.t.

**Abstract**

While Eliashberg and Gromov proved a sharp bound for the minimal target-dimension of proper holomorphic embeddings of Stein-manifolds with dimension $n \geq 2$, this result is to date unknown for Stein-manifolds of dimension one, commonly called Riemann surfaces (Bell-Narasimhan conjecture).

In this talk I will give a short introduction on Riemann surfaces and proper holomorphic embeddings before providing an overview on the currently known examples of properly into $\mathbb{C}^2$ (or large enough subsets) embeddable Riemann surfaces. Furthermore I want to discuss methods of constructing new embeddings and ideas on how to obtain more examples and/or methods.

Katia Winklmaier-Peran (JMU Würzburg)

## Stetigkeit von Gutt-Sternprodukten für Polynome auf dem Kotangentenbündel von Lie-Gruppen

**18. 1. 2019**, 14 Uhr c.t.

**Abstract**

Nach einer Einleitung in die Deformationsquantisierung und in Sternprodukte betrachten wir Gutt-Sternprodukte auf dem Kotangentenbündel von Lie-Gruppen und erarbeiten hierzu eine explizite Formel. Hauptbestandteil meiner Masterarbeit ist dessen Untersuchung auf Stetigkeit. Dazu verwende ich Resultate aus der Masterarbeit von Paul Stapor ("Convergence of the Gutt star product"). Zunächst erfolgt die Untersuchung auf Stetigkeit auf $T^*\mathbb{R}^n$. Darauf aufbauend möchte ich den allgemeinen Fall auf $T^*G$ untersuchen.

Mahdi Hamdan (JMU Würzburg)

## Morita theory for locally convex algebras

**11. 1. 2019**, 14 Uhr c.t.

**Abstract**

Algebraic objects are often examined, by considering their category of representations. So the question arises, whether two algebras have equivalent categories of representations, rather than them being isomorphic. We call two algebraic objects are Morita equivalent, if their categories of representations are. To that extend, Morita theory for purely algebraic objects, like rings, are already well understood. In practice however, one often encounters additional topological properties. In my talk, we will consider the category LCAlg of locally convex algebras. As it is well known from the theory of rings, one can introduce a notion of Morita equivalence by using the notion of a conveniently defined bicategory. By introducing a well behaved tensor product on LCAlg one gets a monoidal category, from which one can construct a bicategory, whose objects being algebras and 1-morphisms being bimodules over these algebras.

Alle zusammen

## Xmas@ChairX

**21. 12. 2018**, 14 Uhr c.t.

**Abstract**

Tobias Schmude (JMU Würzburg)

## Idempotent Completion of Categories and Application to the Theorem of Serre-Swan

**14. 12. 2018**, 14 Uhr c.t.

**Abstract**

In many categories, idempotent endomorphims split, i.e. can be written as the composition of a section with a corresponding retraction. For any category that does not allow this for all idempotents, we construct a universal embedding into a category that does so, called an idempotent completion. For any topological space $X$, we can realize the category $\mathsf{Vect}_X$ of vector bundles over $X$ and the category $\mathsf{Proj}_{\mathscr{C}(M)}$ of projective modules over the continuous functions on $X$ as idempotent completions of easier categories. This yields a conceptual proof of the theorem of Serre-Swan via the universal property of idempotent completions, showing that for many topological spaces $X$ the section functor $\Gamma$ is an equivalence of the above categories.

Jochen Trumpf (Australian National University, Canberra)

## Exploiting symmetry in observer design for flying robots

**7. 12. 2018**, 14 Uhr c.t.

**Abstract**

Many mechanical systems exhibit continuous symmetries that lead to dynamic models for those systems where the state space carries the structure of a homogeneous space of the symmetry Lie group. These models are prevalent in mobile robotics where such symmetries ultimately derive from the physical fact that the laws of rigid body motion are invariant to a change of reference frame. Control theory for systems on Lie groups and homogeneous spaces is a well developed subject that has been studied since the 1970s with several popular textbooks available. In contrast, observer theory for such systems is much less developed and real progress on observer design principles for such systems has only been made in the last 10 years. In this talk I will summarize the current state-of-the-art in observer design for kinematic systems with complete symmetries and show examples of applications of this theory to mobile robotic systems. The resulting state estimation algorithms are computationally much simpler than the alternative stochastic filters but show similar practical stability and robustness properties. Most of the observers in this talk have existing real-time implementations in on-board embedded hardware.

David Kern (JMU Würzburg)

## Lie Algebroids and Unimodular Poisson Manifolds

**30. 11. 2018**, 14 Uhr c.t.

**Abstract**

In the case of symplectic manifolds traces with respect to star products are well-understood. This mainly relies on the existence of Darboux charts. In case of Poisson manifolds this becomes much more difficult. However, Lie algebroids are a nice exception as it is possible to receive traces with respect to homogeneous star products on the dual of a unimodular Lie algebroid. In this talk Lie algebroids are discussed in general first before we define the modular class of a Poisson manifold and mention a way how Lie algebroids could be extended to unimodular Lie algebroids.

Bastian Seifert (JMU Würzburg / Fachhochschule Ansbach)

## Matrix-valued and multivariate Chebyshev polynomials

**23. 11. 2018**, 14 Uhr c.t.

**Abstract**

The classical Chebyshev polynomials of the first and second kind are ubiquitious in applied ma- thematics. Less known and used are their multivariate counterparts. In the representation theory of Lie algebras at least the analogues of the second kind are well-known, as they are the characters of the irreducible representations. In this talk we will show how to construct multivariate Chebyshev po- lynomials for each representation of a Weyl group. In case of the determinant representation one gets the second kind polynomials, i.e. characters. In case of the representation being not one-dimensional one obtains matrix-valued polynomials. After deriving some of their properties we will, if time allows, sketch some of their applications in signal processing and approximation theory.

Thomas Weber (Univ. Neapel)

## Twisted Cartan calculus on smooth submanifolds

**16. 11. 2018**, 14 Uhr c.t.

**Abstract**

If a Lie algebra acts on a smooth manifold by derivations one can use a Drinfel'd twist on the universal enveloping algebra to obtain a deformation quantization of the manifold. In particular the corresponding star product can be expressed in terms of the twist and the action. In this talk I want to extend this idea by not only twist deforming the pointwise product of functions but also the Lie derivative, the insertion derivation, the wedge product and the Lie bracket of vector fields. The deformed structures form a braided Cartan calculus, generalizing the classical Cartan calculus known from differential geometry. Similarly one achieves a twist deformation of the Gerstenhaber algebra structure of multivector fields. Moreover, twist deformed covariant derivatives are treated and I explain why they are the correct notion of covariant derivatives for the twisted Cartan calculus. Afterwards I discuss under which conditions the twisted structures project to smooth submanifolds and I prove that twist deformation and projection commute in those cases. As an example, explicit twist deformation of quadric surfaces is presented. The results are part of joint work with Gaetano Fiore, extending previous works of Paolo Aschieri, Peter Schupp et al.

Gregor Schaumann (JMU Würzburg)

## Quantum field theory from linearising groupoids of $G$-bundles

**9. 11. 2018**, 14 Uhr c.t.

**Abstract**

In this talk we will motivate the framework of a functorial (quantum) field theory and then consider the toy example of a topological gauge theory with finite gauge group $G$ (Dijkgraaf-Witten theory). Its classical fields are principal $G$-bundles and the corresponding quantum field theory is constructed by linearising spans of groupoids. Finally, we outline how to generalize this theory using state sum models.

Alle zusammen

## Vorbesprechung DQ Seminar

**19. 10. 2018**, 14 Uhr c.t.

**Abstract**

# 11 Summer Term 2018

Matthias Schötz (JMU Würzburg)

## The centralizer algebra of abstract $O^*$-algebras

**6. 7. 2018**, 14 Uhr c.t.

**Abstract**

An abstract $O^*$-algebra is a tuple of a $^*$-algebra $\mathscr{A}$ endowed with an order on its Hermitian elements that can be described via a set of positive linear functionals on $\mathscr{A}$. Such abstract $O^*$-algebras can e.g. be used to study the $^*$-representations of locally convex $^*$-algebras or other types of $^*$-algebras. In this talk I will present a general method to construct a new, possibly better-behaved, $^*$-algebra of adjointable left centralizers out of an arbitrary abstract $O^*$-algebra. Special cases of this construction yield a natural way to associate a $C^*$-algebra to every abstract $O^*$-algebra, a reasonable notion of spectrum of elements and hopefully even a spectral theorem for abstract $O^*$-algebras.

Gregor Schaumann (JMU Würzburg)

## From linear algebra to topological field theory

**29. 6. 2018**, 14 Uhr c.t.

**Abstract**

This introductory talk is about a bridge between the geometry of tensor calculus, the Jones polynomial, Hopf algebras and quantum invariants of 3-manifolds. In the end I will describe how recent results about defects and orbifolds in topological field theory, which are joint work with Nils Carqueville and Ingo Runkel, fit into the picture.

Andreas Kraft (JMU Würzburg)

## BRST reduction of quantum algebras with $^*$-involution

**22. 6. 2018**, 14 Uhr c.t.

**Abstract**

In this talk I want to continue the search for involutions on reduced quantum BRST algebras in the context of deformation quantization started in my last talk. Here we saw that the BRST approach in the setting of the classical Marsden-Weinstein reduction leads to an induced star product $*_{\mathrm{red}}$ on the reduced manifold. Moreover, we found a suitable $^*$-involution on the BRST algebra that in the case of cotangent bundles of compact Lie groups led to the complex conjugation as involution for $*_{\mathrm{red}}$. Now I want to consider general manifolds with strongly Hamiltonian actions of compact Lie groups and compatible Hermitian star products and show that we obtain an analogue result. In particular, if the action is in addition free on the regular constraint surface, i.e. if the reduced manifold exists as a smooth manifold, we get again the complex conjugation as an induced $^*$-involution for the reduced star product.

Thomas Bendokat (JMU Würzburg)

## A new Approach to the Essential Manifold

**25. 5. 2018**, 14 Uhr c.t.

### Abstract

We give an explicit formulation of the essential manifold by means of symmetric matrices. Every equivalence class of the fourfold ambiguity is mapped to one point, i.e. one matrix. We also derive formulas for the tangent space, for a Riemannian metric and for geodesics on the essential manifold.

Marvin Dippell (JMU Würzburg)

## A Bicategorical Approach to Morita Equivalence and Reduction

**18. 5. 2018**, 14 Uhr c.t.

### Abstract

In classical mechanics reduction provides a powerful tool to construct out of a given system and an associated symmetry a lower dimensional system with simplified equations of motion. Geometrically, this procedure is given by the Marsden-Weinstein (or more general coisotropic) reduction. Using deformation quantization we can quantize a classical system obtaining a $^*$-algebra describing the observables of our quantum mechanical system. But to get a complete description of the system we need to choose in addition a representation of this $^*$-algebra. Thus knowing the representation theory is an important, but in general not achievable task. Morita theory now provides tools to at least compare the representation theories of different $^*$-algebras. In this talk I will present a framework in which reduction and Morita equivalence can be described for both classical and quantum mechanical systems. From this it will immediately follow that Morita equivalence is preserved under reduction. In addition, I will show that classical limits work well with reduction and give an outlook on some open questions.

Andreas Kraft (JMU Würzburg)

## Involutions for reduced quantum algebras via the BRST approach

**4. 5. 2018**, 14 Uhr c.t.

### Abstract

In deformation quantization, one way to formulate the quantization of a reduced space $M_{\mathrm{red}}$ is the BRST approach. We will stay in the well known setting of the classical Marsden-Weinstein reduction and recall the basics of the classical and quantum BRST algebras. Considering the corresponding cohomologies then leads to a star product $*_{\mathrm{red}}$ on the reduced space that is induced by the reduction.

The new question I will address in the talk is if the BRST reduction gives in addition a natural *-involution for the reduced star product. Therefore, we will look at first for suitable involutions on the BRST algebras and then consider the cotangent bundle of Lie groups as first example.

## Chiara Esposito (JMU Würzburg)

## Quantum momentum map: a beta version

**27. 4. 2018**, 14 Uhr c.t.

**Abstract**

In this talk I will present new results concerning the quantization of momentum map obtained by using the UDF approach proposed by Drinfeld. This opens many new questions that will be further discussed. Finally, we cry all together for my departure.

## Luca Tomassini (Roma)

## Vertex algebras and Conformal Quantum Field Theory

**13. 4. 2018**, 14 Uhr c.t.

**Abstract**

After reviewing some well known facts about Vertex Operator Algebras and one dimensional Conformal Quantum Field Theory, we will present some partial new results on their construction starting from local nets.

Gandalf Lechner (Cardiff)

## Yang-Baxter characters of the infinite symmetric group and subfactors

**9. 3. 2018**, 14 Uhr c.t.

**Abstract**

The Yang-Baxter equation (YBE) is a nonlinear matrix equation that lies at the heart of many subjects, including quantum statistical mechanics, integrable quantum field theory, knot theory, braid groups, subfactors, quantum groups, quantum information ... Due to the nonlinearity and noncommutativity of the YBE, its solutions are notoriously difficult to obtain. In this talk, I will consider involutive solutions R of the YBE ("$R$-matrices", satisfying $R^2 = 1$) and present a complete classification in any dimension. The upshot of this classification is that any involutive $R$-matrix defines a representation and an extremal character of the infinite symmetric group as well as a corresponding tower of inclusions of von Neumann algebras. Using these structures, I will describe how to find all $R$-matrices up to a natural notion of equivalence inherited from applications in QFT (given by the character and the dimension), how to completely parameterize the set of solutions, and how to decide efficiently whether two given $R$-matrices are equivalent. Time permitting, I will also indicate how these results carry over to $q$-deformed $R$-matrices which are no longer involutive.

Francesca Arici (Leipzig MPI)

## A NCG approach to quantum lattice gauge theories and their continuum limit

**9. 3. 2018**, 10 Uhr c.t.

**Abstract**

We will describe the quantization of gauge theories on a graph in terms of their algebras of observables and of the Hilbert space on which the algebra is represented. The algebra of observables for the quantum system admits a natural geometric realization as a groupoid $C^*$-algebra. I will describe the behaviour of such algebras under lattice refinements and the resulting continuum and infinite volume limit of the theory. Based on joint work with R. Stienstra and W. van Suijlekom.

Stephan Huckemann (Institut für Mathematische Stochastik, Universität Göttingen)

## Non-Euclidean Statistics

**8. 3. 2018**, 14 Uhr c.t.

**Abstract**

In order to statistically analyze data on non-Euclidean spaces, such as spheres, shape spaces, or more complicated manifold stratified spaces, we first review some basic statistical hypothesis te- sting and dimension reduction techniques. Then we discuss intricacies of non-Euclidean central limit theorems, non-Euclidean principal component methods and see, among others, that some elementary spaces are variably benevolent towards this later task, calling for deformation of canonical geometry. Finally, we discuss hypothesis testing in a very general non-Euclidean setting and we illustrate the theory presented with applications to RNA structure analysis and adult stem cell diversification. We conclude with a selection of challenges in Non-Euclidean Statistics.

Chiara Esposito (JMU Würzburg)

## Quantum momentum map via twist

**9. 2. 2017**, 14 Uhr c.t.

**Abstract**

In this talk I will introduce the notion of Hamiltonian action in the setting of Poisson Lie groups actions. This notion can be easily extended to the setting of Hopf algebra actions, allowing a quanti- zation via Drinfeld twist.

Maximilian Hanusch (JMU Würzburg)

## The regularity problem for Milnor's infinite dimensional Lie groups

**2. 2. 2018**, 14 Uhr c.t.

**Abstract**

he right logarithmic derivative and its inverse – the generalized integral – play a central role in Lie theory. For instance, existence of the exponential map – indispensable for structure theory of Lie groups – is equivalent to integrability of each constant curve. Similarly, existence of holonomies – essential for gauge field theories – is based on integrability of pairings of connections with derivatives of curves in the base manifold. The most important questions to be clarified in the infinite dimensional context are (1.) under which circumstances is a given Lie algebra-valued curve integrable, and (2.) presumed that each (Lie algebra-valued) $C^k$-curve is integrable, under which circumstances is the evolution map smooth w.r.t. the $C^k$-topology. Both issues turn out to be primarily of topological nature as closely related to the continuity properties of the Lie group multiplication as well as to the completeness properties of the Lie group and its modeling space.

Matthias Schötz (JMU Würzburg)

## On pure states and characters of $^*$-algebras

**12. 1. 2018**, 14 Uhr c.t.

**Abstract**

It is easy to see that every character (i.e. unital -homomorphism to $\mathbb{C}$) of a commutative unital associative $^*$-algebra is a pure state (i.e. extreme point in the convex set of all normalized algebraically positive linear functionals). In this talk I am going to discuss sufficient conditions for the converse to be true as well. In order to formulate these results together with similar ones, e.g. for locally convex $^*$-algebras, the notion of an abstract $O^*$-algebra (unital associative $^*$-algebra with an order defined by positive linear functionals) is introduced.

All together

## Xmas@ChairX

**15. 12. 2017**, 14 Uhr c.t.

**Abstract**

Thomas Bendokat (JMU Würzburg)

## Encoding camera positions in a Riemannian manifold

**8. 12. 2017**, 14 Uhr c.t.

**Abstract**

Reconstructing the positioning of two cameras at different locations from visual data can be done by using essential matrices. We give an introduction about what essential matrices are and then show how they form a Riemannian quotient manifold. We compare different ways of describing this manifold and their respective advantages and disadvantages, and give an outlook towards a suitable matrix description as well as its use for geodesics, parallel transport and curvatures.

Marvin Dippell (JMU Würzburg)

## Morita equivalence and reduction

**1. 12. 2017**, 14 Uhr c.t.

**Abstract**

In classical mechanics the famous theorem of Noether tells us that symmetries can be used to simplify the equations of motion of a given system. Geometrically a classical mechanical system is modeled as a symplectic manifold and simplifying the equations of motion corresponds to the so called Marsden-Weinstein reduction, or more general to coisotropic reduction. Using deformation quantization we can quantize a classical system obtaining a $^*$-algebra describing the observables of our quantum mechanical system. But to get a complete description of the system we need to choose in addition a representation of this $^*$-algebra. Thus knowing the representation theory is an important, but in general not achievable task. Morita theory now provides tools to at least compare the representation theories of different $^*$-algebras. Taking into account reduction on the quantum side leads us to the main question of the talk: Does reduction preserve Morita equivalence? To formulate this question precisely I will present the basics of Morita theory and of different reduction procedures, and in the end give first ideas on how this problem may be solved using a quantum analogue of coisotropic reduction.

Bastian Seifert (HS-Ansbach und JMU Würzburg)

## FFT algorithms based on algebraic induction

**24. 11. 2017**, 14 Uhr c.t.

**Abstract**

Algebraic signal processing theory gives a unified setting for various linear signal processing mo- dels. We first give an introduction to the basic concepts like algebraic signal models and corresponding Fourier transforms. Then we show how one can use the language of algebraic geometry and algebraic induction to derive fast algorithms for the Fourier transform of an algebraic signal model. As an example we show how one can derive the classical Cooley-Tukey FFT. We give sufficient conditions for a set of multivariate polynomials to give rise to fast algorithms.

## Francesco D'Andrea (Napoli)

## A tutorial on graph $C^*$-algebras

**17. 11. 2017**, 14 Uhr c.t.

**Abstract**

Many well-known examples of quantum spaces belong to a class of $C^*$-algebras associated to row-finite directed graphs, or graph $C^*$-algebras for short. This talk will be a guided tour on graph $C^*$-algebra, from the point of view of noncommutative geometry. I will recall the definition, give a number of examples and discuss some of the main properties of such $C^*$-algebras.

## Lukas Miaskiwskyi (JMU Würzburg)

## Group averages: Intuition and examples

**10. 11. 2017**, 14 Uhr c.t.

**Abstract**

Motivated by invariant Hochschild cohomology, we investigate the concept of averaging an object by a group action, why the name "average" is justified in this context and when taking such an average is even possible. To gain intuition about group actions, we start with simple examples to understand the properties that make a group action well-behaved. Different averaging procedures for differently behaving actions are introduced and interpreted. If enough time is left, the averaging procedures with their applications to invariant Hochschild cohomology are discussed.

## Jean Gutt (Universität Köln)

## Knotted symplectic embeddings

**3. 11. 2017**, 14 Uhr c.t.

**Abstract**

I will discuss a joint result with Mike Usher, showing that many toric domains $X$ in the 4-dimensional euclidean space admit symplectic embeddings $f$ into dilates of themselves which are knotted in the strong sense that there is no symplectomorphism of the target that takes $f(X)$ to $X$.

## Matthias Schötz (JMU Würzburg)

## The geometry of classical field theories

**27. 10. 2017**, 14 Uhr c.t.

**Abstract**

Before trying to quantize field theories, it seems reasonable to get a good understanding of the classical formulation of such theories. Like for classical mechanics, the classical field theories should be treated with the help of (differential-)geometry, which allows to understand which structures arise naturally (like Poisson brackets) and how to perform phase space reduction. Unlike for

classical mechanics, however, the theory of geometric field theories is still far from being complete. In this talk I will first present the basics of the Lagrangian and covariant Hamiltonian formalism for field theories, where the tangent and cotangent space of a manifold are replaced by the jet and cojet bundle of a fibre bundle. Then I am going to comment on some resent developments concerning Poisson brackets and on some seemingly open problems that might be of interest for future research.

All together

## Organization meeting

**20. 10. 2017**, 14 Uhr c.t.

**Abstract**

# 13 Summer Term 2017

Chiara Esposito (JMU Würzburg)

## Formality and Poisson Actions

**27. 7. 2017**, 14 Uhr c.t.

**Abstract**

Florian Ullrich

## Rolling Maps for Real Stiefel Manifolds II

**21. 7. 2017**, 14 Uhr c.t.

**Abstract**

In continuation of last week, this talk aims at pursuing the examination of the real Stiefel manifold $St_{n,k}$ and its rolling motion on affine tangent spaces. Especially, certain advantageous applications are the focus of our concern. Primarily, we will exploit a particular 1-1-correspondence with straight lines on the considered tangent space to derive a closed formula expressing geodesics on $St_{n,k}$. In accordance, a *geodesic equation* describing geodesics on $St_{n,k}$ in general will be derived by variational methods. We moreover want to construct a crucial bond between the theory of rolling motions and the realization of parallel transport. Finally, an observation in terms of controllability of the considered control system will be briefly presented.

Florian Ullrich

## Rolling Maps for Real Stiefel Manifolds I

**14. 7. 2017**, 14 Uhr c.t.

**Abstract**

In this talk I want to examine the real Stiefel manifolds $St_{n,k}$ rolling on an affine tangent space without slipping or twisting. Correspondingly, this talk firstly summarizes fundamental constructions and essential techniques. Hence, presenting a brief overview of $St_{n,k}$ and its indispensable properties will be one of the crucial points, especially verifying the structure of homogeneous spaces. In conclusion, I want to introduce an extrinsic formalism for describing certain rolling motions of smooth manifolds. It was firstly established by Richard W. Sharpe in 1997. Next week's talk continues these considerations by various applications in Riemannian geometry.

Lukas Miaskiwskyi

## $G$-invariant Hochschild cohomology and existence of a HKR map

**7. 7. 2017**, 14 Uhr c.t.

**Abstract**

In this talk, first some motivation for Hochschild cohomology is given in the context of deformation quantization, and the notion of group invariance of this cohomology is introduced. Applying the usual Hochschild-Kostant-Rosenberg (HKR) isomorphism does not restrict to the $G$-invariant case without difficulties, leaving the space without immediate geometric interpretation. This and possible solutions will be discussed. Once the restriction of HKR turns out to be meaningful, some language from principal bundle theory will be used to give rise to a geometric understanding of this $G$-invariant cohomology, which may be highly valuable for reduction of physical systems.

Jonathan Engle (Florida Atlantic University)

## Uniqueness of the Representation in Homogeneous Isotropic LQC

**26. 6. 2017**, 14 Uhr c.t.

### Abstract

We show that the standard representation of homogeneous isotropic loop quantum cosmology (LQC) used in the literature is the GNS-representation associated to the unique state on the reduced quantum holonomy-flux $*$-algebra that is invariant under residual diffeomorphisms – *both* when the standard algebra with holonomies along only straight edges is used, as well as when one extends the algebra to include curved edges (a la Fleischhack). In order for the residual diffeomorphisms to have a well-defined action on the quantum algebra, we have let them act on the fiducial cell as well as on the dynamical variables, thereby recovering covariance.

Martin Bordemann (Mulhouse)

## Combinatorics of covariant derivatives and Duflo-type star-products

**23. 6. 2017**, 14 Uhr c.t.

### Abstract

In 1969, Michel Duflo has shown that for any finite-dimensional Lie algebra g the invariants of the symmetric algebra generated by g (under the adjoint representation) and the centre of the universal enveloping algebra of g are isomorphic as commutative algebras by means of an explicit isomorphism. Since the former can be interpreted as a subalgebra of covariantly constant symmetric tensor fields on a corresponding Lie group, it is interesting in general to study subalgebras of differential operators whose standard symbols w.r.t a covariant derivative are covariantly constant, and check whether they are commutative and isomorphic to their pointwise multiplications. We present an algebraic way to attack the problem by studying iterated covariant derivatives and their combinatorics using the Hopf structure of free algebras, i.e. the shuffle comultiplication, primitive elements etc. in the framework of anchored modules, Lie-Rinehart algebras and Rinehart bialgebras.

Philipp Schmitt

## Construction of a Locally Convex Topology with respect to which Karabegov's Star Product on Spheres is continuous

**24. 5. 2017**, Sondertermin: 10 Uhr c.t.

### Abstract

The idea of deformation quantization is to deform a commutative algebra of observables into a non-commutative one. Usually, deformation quantization refers to formal deformations, i.e. it gives the product in terms of power series of a formal parameter. In physical applications the formal parameter needs to be substituted with a real number, the specific value of Planck's constant, leading to the question whether the formal series converge for a suitable subalgebra of observables.

Sometimes it is easy to write down such a subalgebra, e.g. a subalgebra for which the formal power series have only a finite number of non-zero terms. These subalgebras are usually rather „small“. Hence one would like to take their completion with respect to a topology for which the star product is continuous, since, in this case, the star product extends uniquely to the completion. The construction of such a topology for a star product on spheres will be the topic of this talk.

First, I want to recall a construction of star products on coadjoint orbits, which is due to Karabegov. This construction works for all coadjoint orbits of compact, semi-simple, connected and simply-connected Lie groups. Fixing a complex structure on the coadjoint orbit, we can always construct a star product of Wick type. The star product of two fixed functions is a rational function of the deformation parameter $\hbar$, that may have a finite number of poles.

I want to treat the particular case of coadjoint orbits for $SU(2)$, which are spheres. In the case of a unit sphere, poles can only occur for $\hbar = \frac{1}{n}$ with $n \in \mathbb{N}$. I will introduce a locally convex topology, called the $T_R$ topology, on $\mathrm{Pol}(\mathfrak{su}_2)$ and construct a topology on the polynomials on the coadjoint orbit by taking a quotient. With explicit formulas, that express Karabegov's star product as a deformation of the Gutt star product on the Lie algebra $\mathfrak{su}_2$, one can prove that Karabegov's star product is continuous with respect to this quotient topology for $\hbar \neq \frac{1}{n}$.

If time permits, I will say a few words on generalizing this result to coadjoint orbits of arbitrary compact, semi-simple, connected and simply-connected Lie groups.

Matthias Schötz (JMU Würzburg)

## Introduction to $O^*$-algebras

**12. 5. 2017**, 14 Uhr c.t.

### Abstract

$O^*$-algebras are $^*$-algebras realized by (in general unbounded) closable operators on a Hilbert space, or equivalently, by adjointable endomorphisms of a pre-Hilbert space. This talk will discuss some basics of the theory of unbounded operators and $O^*$-algebras, especially the properties of various natural topologies on such algebras, and how $O^*$-algebras arise as representations in non-formal deformation quantisation.

Thomas Weber (University of Naples "Federico II")

## Morita vs Drinfel'd – The Empire Twists Back

**5. 5. 2017**, 14 Uhr c.t.

### Abstract

It is well-known, that Morita equivalent algebras possess equivalent categories of modules. In particular, one is able to compare their representation theories. For this, Morita equivalence is a powerful tool to control the states of a physical system. Moreover, it is an interesting theory by itself, including techniques of finitely generated projective modules. In this talk, we focus on Morita equivalence of a particular class of star product algebras. Namely those, which are related to Drinfel'd twists, i.e. Deformation Quantizations induced by symmetries. After introducing the basic notions and results, we demonstrate, that in this situation Morita equivalence already implies equivalence of star products. In particular, non-trivial Morita classes obstruct the existence of twist star products. Remark, that we have to assume a mild lifting property of the module action. Since Morita equivalence is characterized by the action of the Picard group, we obtain Chern classes of line bundles over the manifold as obstructions for twist star products. If there is time we conclude by discussing some examples.

All together

## Organization meeting

**28. 4. 2017**, 14 Uhr c.t.

**Abstract**

## 14 Winter Term 2016/2017

Michael Forger (Sao Paulo)

Symmetries in Geometric Field Theory and Lie Groupoids

**3. 2. 2017**, 14 Uhr c.t.

**Abstract**

Henrique Bursztyn

Morita equivalence of formal Poisson structures

**Donnerstag, den 26. 1. 2017**, 10 Uhr c.t.

**Abstract**

Michael Forger (Sao Paulo)

The $C^*$-Algebra of the Canonical Commutation Relations

**13. 1. 2017**, 14 Uhr c.t.

**Abstract**

All together

Xmas@ChairX

**16. 12. 2016**, 14 Uhr c.t.

**Abstract**

Niek de Kleijn (ULB)

An equivariant algebraic index theorem

**15. 12. 2016**, 14 Uhr c.t.

**Abstract**

The algebraic index theorem (Fedosov, Nest-Tsygan 1995) is an algebraic counterpart to the well-known and celebrated Atiyah-Singer index theorem. Starting from the basic observation that the pseudo-differential operators on a manifold $X$ form a deformation quantization of the cotangent bundle of $X$ one generalizes the "algebraic part" of the A-S index theorem to a theorem for any formal deformation quantization of any symplectic manifold. In this talk we will formulate and discuss an equivariant version of the algebraic index theorem where one considers also the action of a discrete group on the deformation quantization. More specifically we show how one can compute a canonical trace on the crossed product of the deformation quantization algebra with the discrete group in terms of characteristic classes on the homotopy orbit space.

Paul Stapor (Helmholtz-Zentrum München)

## Scalable Methods for Parameter Estimation in Systems Biology – Blessing or Curse?

**1. 12. 2016**, 14 Uhr c.t.

**Abstract**

In this talk, we briefly present the concept of parameter estimation for mechanistic models in systems biology. The main part of this field consists of a setting up and solving a dynamical system (e.g. an ODE, PDE or DAE), which describes a certain biological process. Those dynamical systems usually depend on many parameters (e.g. reaction rates), for which no numerical values are known. Hence, an inverse problem must be solved, in order to estimate those parameter from measurement data. The main point in this context is, the bigger the mathematical models are, the more the solution of this inverse problem becomes computationally expensive. Hence, computationally efficient methods are an important field of study.

Chiara Esposito (JMU Würzburg)

## Formality and Momentum map

**25. 11. 2016**, 14 Uhr c.t.

**Abstract**

In this talk we discuss ideas for classification of quantum actions and quantum momentum map. In particular, we give basic notion of momentum map in the setting of Poisson Lie actions and we recall the main ideas of formality theory. We conjecture that formality theorem for cochains classifies the quantum actions and formality for chains classifies quantum momentum maps.

Michael Forger

## Symmetrisch hyperbolische Systeme von Differentialgleichungen und kausale Strukturen

**18. 11. 2016**, 14 Uhr c.t.

**Abstract**

Wir stellen einen erweiterten Begriff der symmetrischen Hyperbolizität für Systeme von (linearen) partiellen Differentialgleichungen erster Ordnung vor, der — im Gegensatz zur ursprünglichen Version von Friedrichs — den Dirac-Operator einschließt und für den die fundamentale Eigenschaft der Wohlgestelltheit des Cauchyschen Anfangswertproblems nach wie vor garantiert ist. Es zeigt sich, dass jedes solche System eine kausale Struktur auf der unterliegenden Mannigfaltigkeit definiert, die jedoch wesentlich allgemeiner sein kann als die kausalen Strukturen, die von einer Lorentz-Metrik herrühren.

Chiara Pagani (Georg-August-Universität Göttingen)

## A sheaf theoretic description of quantum principal bundles

**11. 11. 2016**, 14 Uhr c.t.

### Abstract

I will describe an algebraic approach to the study of principal bundles based on sheaves of Hopf- Galois algebra extensions. I will report on a general theory of deformation via Drinfeld twists of (sheaves of) Hopf-Galois extensions and show how to construct in this way examples of quantum principal bundles. A twist on the Hopf algebra of the structure group of the bundle generates a new bundle with twisted fibers. A twist on a group of bundle automorphisms generates a bundle with twisted base space.

Based on joint works with L. Cirio and with P. Aschieri, P. Bieliavsky and A. Schenkel.

Julian Keller (JMU Würzburg)

## Mathematical Aspects of the Toda Lattice

**4. 11. 2016**, 14 Uhr c.t.

### Abstract

This talk aims to give an overview over some aspects of the finite Toda lattice, especially the non- periodic one. In particular, we'll use Lax pairs to obtain integrals of motion and show that the Toda lattice is a completely integrable Hamiltonian system. Following this, we'll see how the non-periodic Toda lattice can be regarded as dynamical system on a coadjoint orbit of the group of triangular matrices and how the classical r-matrix approach can be used to show the complete integrability of the non-periodic Toda lattice using only group theoretical methods.

Iakovos Androulidakis (University of Athens)

## Almost regular Poisson manifolds and their holonomy groupoids

**27. 10. 2016**, 10 Uhr c.t.

### Abstract

We introduce a big class of Poisson manifolds, the almost regular ones. Roughly, they are the Poisson manifolds whose symplectic foliation is regular in a dense open subset. All regular Poisson manifolds are included in this class, as well as all the log-symplectic manifolds and certain Heisenberg- Poisson manifolds. We are looking for desingularizations of such structures, which means a Poisson groupoid which defines the symplectic foliation and whose Poisson structure is regular. A natural candidate is the associated holonomy groupoid, which is smooth in this category. We show that, moreover, this groupoid satisfies all our other requirements. In the case of log-symplectic manifolds, a simple minimality argument shows that it coincides with the symplectic groupoid constructed by Gualtieri and Li. And for the Heisenberg-Poisson manifolds under consideration, it is exactly Connes' tangent groupoid. This hints that various blow-up constructions in Poisson geoemtry can be replaced by the systematic construction of the holonomy groupoid of a singular foliation.

Philipp Schmitt

## Construction of Star Products on Coadjoint Orbits II

**21. 10. 2016**, 14 Uhr c.t.

### Abstract

In this talk I want to present a particular construction of a star product on coadjoint orbits due to Karabegov. It is known that for a semisimple Lie group, the Gutt star product doesn't restrict to coadjoint orbits, necessitating in a more complicated construction. We will introduce a subspace of the space of functions on the Lie group, that is also important for Harish Chandra theory. We may turn this space into a noncommutative algebra by giving a bijection to a quotient of the universal enveloping algebra. Furthermore it turns out that under certain conditions we can push forward this algebra to functions on the coadjoint orbit, yielding the desired star product.

Philipp Schmitt

## Construction of Star Products on Coadjoint Orbits

**14. 10. 2016**, 14 Uhr c.t.

### Abstract

In this talk I want to present a particular construction of a star product on coadjoint orbits due to Karabegov. It is known that for a semisimple Lie group, the Gutt star product doesn't restrict to coadjoint orbits, necessitating in a more complicated construction. We will introduce a subspace of the space of functions on the Lie group, that is also important for Harish Chandra theory. We may turn this space into a noncommutative algebra by giving a bijection to a quotient of the universal enveloping algebra. Furthermore it turns out that under certain conditions we can push forward this algebra to functions on the coadjoint orbit, yielding the desired star product.

All together

## Organizational Meeting

**7. 10. 2016**, 14 Uhr c.t.

### Abstract

## 15 Summer Term 2016

Pierre Bieliavsky (UC Louvain-la-Neuve)

### A smooth non-commutative hyperbolic plane

**15. 7. 2016**, 14 Uhr c.t.

**Abstract**

I'll present a construction of a non-formal star-product algebra on the hyperbolic plane $D := SL(2, \mathbb{R})/SO(2)$. The main features are the following ones.

- Smoothness: the star-product closes on a Fréchet nuclear space $S$ of smooth functions on $D$ analogue to the Schwartz space in the Weyl-Moyal quantization. In particular, $S$ densely contains the space of test functions on $D$, is closed under both pointwise and deformed multiplication as a Fréchet algebra. The group $SL(2, \mathbb{R})$ naturally acts in a smooth way on each of the resulting one-parameter family of algebras.
- Representability: every such deformed algebra structure equivariantly represents as compact operators on (projective) holomorphic discrete series.
- Traciality: every deformed algebra structure on $S$ uniquely extends to the space of square-integrable function classes on $D$ as a tracial Hilbert algebra that unitarily represents the Hilbert-Schmidt operator ideal on the corresponding holomorphic discrete series representation.

Matthias Schötz (JMU Würzburg)

### From $^*$-Algebras to uniform Spaces and back

**8. 7. 2016**, 14 Uhr c.t.

**Abstract**

The Gel'fand transformation allows one to represent commutative (unital) $C^*$-algebras as algebras of continuous functions on a compact space. While this construction can be carried out for arbitrary commutative $^*$-algebras, its properties are way less interesting in the general case. However, one can still look for sufficient conditions for the Gel'fand transformation to have good properties (e.g. being a faithful representation). In this talk, the following setting will be examined: Let $\mathscr{A}$ be a unital associative $^*$-algebra, $\Phi \subseteq \mathscr{A}^*$ a cone of "interesting" positive linear functionals (e.g. the continuous ones in the case that $\mathscr{A}$ carries a topology) and $\beta \subseteq P(\mathscr{A})$ a bornology on $\mathscr{A}$ with respect to which all $\varphi \in \Phi$ are bounded. After a short presentation of some general properties of these objects, I will give a version of the Gel'fand transformation that represents such algebras by algebras of uniformly continuous functions on a uniform space and discuss some conditions for this representation to be faithful. Conversely, the $^*$-algebra of bounded complex functions on a complete uniform space can be equipped in a canonical way with a cone of positive linear functionals and a bornology, such that the Gel'fand transformation yields the original uniform space.

Jonas Schnitzer

### An Universal Construction of Universal Deformation Formulas and Drinfel'd Twists

**1. 7. 2016**, 14 Uhr c.t.

**Abstract**

Given an associative Algebra $\mathcal{A}$ with a Poisson bracket induced by a $r$-matrix, we want to understand quantisations of this bracket. It turns out, that we can use the well-known Fedosov construction, in a slightly modificated way, to give a formula for such a quantisation. On the other hand, we can construct a Drinfel'd twist with the same method, which induces a quantisation for this algebra as well. The comparison of the two approaches shows that that the two deformations of $\mathcal{A}$ coincide.

Francesca Spagnuolo (University of Valencia)

## On a class of large groups that controls some embedding properties

**24. 6. 2016**, 14 Uhr c.t.

**Abstract**

Frederik vom Ende

## Dilation of Completely Positive Maps

**17. 6. 2016**, 14 Uhr c.t.

**Abstract**

Operationen auf offenen quantenmechanischen Systemen werden durch sogenannte Quantenkanäle beschrieben, welche lineare, spur-erhaltende und vollständig positive Abbildungen sind. Wir wollen verstehen, warum die Bedingung der vollständigen Positivität notwendig ist, welche Struktur diese mit sich bringt und warum sich jeder Quantenkanal als unitäre Operation mit anschließender partieller Spurbildung darstellen lässt.

Bastian Seifert (HS-Ansbach und JMU Würzburg)

## Algebras, Signals and Algorithms

**10. 6. 2016**, 14 Uhr c.t.

**Abstract**

In this talk we will give a short introduction to the algebraic signal processing theory of Püschel and collaborators, where we will focus on how to build FFT-like algorithms for transformations from time/space domain to frequency domain using polynomial algebras. Then we give an introduction to multivariate Chebyshev polynomials, which are suitable for developing such algorithms on lattices associated to root systems.

Alfonso G. Tortorella (University of Firenze)

## Deformations of coisotropic submanifolds of Jacobi manifolds

**3. 6. 2016**, 14 Uhr c.t.

**Abstract**

In this paper, using the Atiyah algebroid and first order multi-differential calculus on non-trivial line bundles, we attach an $L_\infty$-algebra to any coisotropic submanifold $S$ in an abstract (or Kirillov's) Jacobi manifold. Our construction generalizes and unifies analogous constructions in symplectic, Poisson case, and locally conformal symplectic geometry. As a new special case, we attach an $L_\infty$-algebra to any coisotropic submanifold in a contact manifold, including Legendrian submanifolds. The $L_\infty$-algebra of a coisotropic submanifold $S$ governs the (formal) deformation problem of $S$.

Matthias Schötz (JMU Würzburg)

## An unusual power series expansion for certain holomorphic functions

**27. 5. 2016**, 14 Uhr c.t.

**Abstract**

Let $\overline{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ be the Riemann-sphere,

$$\Omega := \left(\overline{\mathbb{C}} \times \overline{\mathbb{C}}\right) \setminus \left(\left\{(x,y) \in \mathbb{C} \times \mathbb{C} \;\middle|\; xy = 1\right\} \cup \{(0,\infty),(\infty,0)\}\right)$$

and denote by $\mathscr{O}(\Omega)$ the Fréchet space of all holomorphic functions on $\mathscr{O}(\Omega)$ with the usual topology of uniform convergence on all compact subsets of $\Omega$. I will prove (or sketch the prove) that all $\hat{f} \in \mathscr{O}(\Omega)$ can be represented by a locally-uniformly and absolutely converging power-series

$$\hat{f}(x,y) = \sum_{p,q=0}^{\infty} f_{p,q}\hat{e}_{p,q}(x,y) \qquad \text{for all } (x,y) \in \Omega \tag{$*$}$$

with $\hat{e}_{p,q}(x,y) = \frac{x^p y^q}{(1-xy)^{\max\{p,q\}}}$ for $(x,y) \in \Omega \cap \mathbb{C}^2$. More precisely, let $\mathscr{A} \subseteq \mathbb{C}^{\mathbb{N}_0 \times \mathbb{N}_0}$ be the subspace of all series fulfilling $\|f\|_R < \infty$ for all $R \in \mathbb{R}^+$, where

$$\mathscr{A} \ni f \;\mapsto\; \|f\|_R := \sum_{p,q=0}^{\infty} |f_{p,q}| R^{p+q} \in [0,\infty],$$

and endow $\mathscr{A}$ with the locally convex topology of all these seminorms $\|\cdot\|_R$ . Then mapping a series $f \in \mathscr{A}$ to $f \in \mathscr{O}(\Omega)$ like in $(*)$ is an isomorphism of Fréchet spaces.

Knut Hüper

## Applying Differential Geometry to Problems in Control, Optimization, Signal Processing, Interpolation and Statistics

**20. 5. 2016**, 14 Uhr c.t.

**Abstract**

It is well known that differential geometry has found its way into physics already for ages. However, it is more recent that concepts from Lie theory or differentiable manifolds are considered useful tools for e.g. improving the efficiency of numerical algorithms or even for the design of new procedures in the above areas of applied mathematics.

## Chiara Esposito

# Reduction of pre-Hamiltonian actions

**20. 5. 2016**, 16 Uhr c.t.

**Abstract**

I will present a recent reduction theorem for the tangent bundle of a Poisson manifold endowed with a pre-Hamiltonian action of a Poisson Lie group. In the special case of a Hamiltonian action of a Lie group, we are able to compare our reduction to the classical Marsden-Ratiu reduction.

## Thomas Weber (JMU Würzburg)

# No Twist

**13. 5. 2016**, 14 Uhr c.t.

**Abstract**

## Jonas Schnitzer

# A simple algebraic construction

**6. 5. 2016**, 14 Uhr c.t.

**Abstract**

This talk is meant to present an alternative way of proving the Drinfel'd theorem, which states that for a given *r-matrix* there is a *Drinfel'd twist*.

In 1983 Drinfel'd provided a pure existence proof using cohomological arguments which makes it almost impossible to compute examples of these twists. We use a different technique, namely a modified Fedosov-like construction, which is (up to recursion formulas) constructive.

## Paul Stapor (JMU Würzburg)

# Topologies on universal enveloping Algebras

**22. 4. 2016**, 14 Uhr c.t.

**Abstract**

It is known that for certain (possibly infinite dimensional) locally convex Lie algebras $\mathfrak{g}$, their universal enveloping algebras can be topologized by a functorial construction. For finite-dimensional Lie algebras, this construction is optimal, if one wants a large completion of $\mathrm{U}(\mathfrak{g})$. In this talk, we will see how to adapt the construction of this topology in order to show, that it is even optimal for Banach-Lie algebras. Moreover, given a Banach-Lie algebra g, we will present a necessary and sufficient criterion for locally convex topologies on $\mathrm{U}(\mathfrak{g})$, which allow group-like elements in their completions.

Thorsten Reichert (JMU Würzburg)

# Symmetry Reduction in Deformation Quantization and the Kirwan Map

**15. 4. 2016**, 14 Uhr c.t.

**Abstract**

Given a Marsden-Weinstein reduced symplectic manifold, one can, by BRST-cohomological me- thods, construct a map from star products with quantum momentum maps (equivariant star products) on the unreduced manifold to star products on the reduced one. Seeing that equivariant star products are classified by equivariant cohomology and that star products are classfied by de Rham cohomology, we will be interested in the induced map of the reduction map on cohomology. It will turn out to be an analogue of the well-known Kirwan map, but on the cohomology of the Cartan model of equivariant cohomology.

All together

### Rigidity

**5. 2. 2016**, 14 Uhr c.t.

**Abstract**

Matthias Schötz (JMU Würzburg)

### Introduction to Riesz spaces

**22. 1. 2016**, 14 Uhr c.t.

**Abstract**

I will give an introduction to (Dedekind complete) Riesz spaces, i.e. ordered real vector spaces in which all infima and suprema of finite (of all bounded) non-empty subsets exist. Topics will be some important examples (spaces of continuous / uniformly continuous functions, spaces of certain integ- rable functions, the real subspace of hermitian operators in a commutative von Neumann algebra), the Yosida representation theorem, the Freudenthal spectral theorem and how the spectral theorem for bounded linear operators on a Hilbert space can be proven with Riesz-space technology.

Chiara Esposito (JMU Würzburg)

### Rigidity and path method

**15. 1. 2016**, 14 Uhr c.t.

**Abstract**

In this talk we show some basic facts about rigidity of group actions and we give a proof for rigidity of compact symplectic group actions on a compact symplectic manifold which uses the path method. Furthermore, we will show how to generalize this approach to Poisson Lie actions on $b$-Poisson manifolds.

All together

### HNY and planing of next semester

**8. 1. 2016**, 14 Uhr c.t.

**Abstract**

### All together
## Xmas@ChairX
**18. 12. 2015**, 14 Uhr c.t.

**Abstract**

### Thorsten Reichert (JMU Würzburg)
## Introduction to Synthetic Differential Geometry
**11. 12. 2015**, 14 Uhr c.t.

**Abstract**

Following the well known "Pizza Seminar" talk of Michael Shulman on the topic, we will give an introduction to the very basic ideas and concepts of synthetic differential geometry. Starting off with an "axiomatic wishlist" on differential calculus, we will introduce nilsquare infinitesimals, that is elements of a field which square to zero, and explore their implications. As is to be expected, there are various problems involving the naive definition of nilsquares and we will have to look for ways to deal with them (which will include a short discourse into constuctive logic). Finally, we will use those concepts to set up geometry on certain (microlinear) spaces, including a discussion on tangent vectors and their properties.

### Matthias Schötz (JMU Würzburg)
## An algebra of analytic functions on the Poincaré disc
**4. 12. 2018**, 14 Uhr c.t.

**Abstract**

The Poincaré disc has been discussed in the context of strict deformation quantisation from various points of view. For example, Svea Beiser constructed a Fréchet topology on a space of analytic functions on the disc under which a certain star-product is continuous. There is also a work by Pierre Bieliavsky and Victor Gayral where they construct a $C^*$-algebraic deformation of the disc. In order to compare the two approaches we need a better understanding of the action of the group $SU(1,n)$ on the Poincaré disc and of the properties of the space of analytic functions considered by S. Beiser. In this talk, I will outline some aspects of the representation theory of $SU(1,n)$ on the disc $\mathbb{D}$ and discuss a space of functions on $\mathbb{D}$ that can be expressed as $f(z) = \hat{f}(z, \overline{z})$ for all $z \in \mathbb{D}$, where $f$ is a function holomorphic on a certain subset $\Omega \subseteq \mathbb{C} \times \mathbb{C}$ containing $\mathbb{D} \times \mathbb{D}$. It turns out that this space of functions is most probably the space of analytical functions where the approach of S. Beiser works.

### Stefan Waldmann
## Some Ideas on Group Algebras
**27. 11. 2015**, 14 Uhr c.t.

**Abstract**

In this talk I will present some joint work with Michel Cahen and Simone Gutt on topologies for group algebras of finitely generated groups. Based on concepts of geometric group theory the topologies use length functionals on the group, depending on choices of generating sets. While the seminorms actually depend on these choices, the resulting topologies do not. Several growth properties of the group are then encoded in locally convex analytical properties of the topologies like nuclearity.

Marco Benini (Universität Potsdam)

## Differential cohomology with compact support

**20. 11. 2015**, 14 Uhr c.t.

**Abstract**

Starting form a relative version of Cheeger-Simons differential characters, we introduce differential cohomology with compact support by a natural limiting prescription. It turns out that the resulting notion of compactly supported differential cohomology fits into a 3-by-3 diagram of short exact sequences similar to the one for ordinary differential cohomology, however all objects are replaced by their compactly supported analogues; in particular, we interpret compactly supported differential cohomology as a refinement of compactly supported integral cohomology by differential forms with compact support. We show that differential cohomology with compact support is naturally isomorphic to the (smooth) Pontryagin dual of ordinary differential cohomology. As an application, the Cauchy problem relevant to (higher) Abelian duality is solved for arbitrary and spacelike compact configurations.

Jonas Schnitzer

## The deformed Koszul complex

**6. 11. 2015**, 14 Uhr c.t.

**Abstract**

The deformed Koszul complex is a very powerful tool in the theory of quantization of submanifolds of a already quantized manifold. After discussing the obstructions of a star product to be shrinkable to a submanifold and the so-called tangential star products, we will briefly develop the new technology, namely the Koszul complex, try to deform it and discuss the obstructions to do so.

Afterwards we try to understand what happens to the star product on the big manifold. It turns out that we force the star product to be indeed tangential to the submanifold, so we can canonically shrink it.

To show that this is not an empty theory, we will shortly discuss a big class of examples, namely the compact Lie groups with regular coadjoint orbits.

Joakim Arnlind (Linköping)

## Riemannian curvature of the noncommutative 3-sphere

**28. 10. 2015**, 14 Uhr c.t.

**Abstract**

Understanding the Riemannian curvature of noncommutative manifolds is a wide open problem. Although there has been quite some activity during the last few years, it is far from clear if there exists a full Riemann curvature tensor or how one should find it. There is a “natural” analytic definition of the scalar curvature in terms of the expansion of the heat kernel, and the scalar curvature has been computed for noncommutative tori. In this talk I will give a pedestrian approach to Riemannian curvature on the noncommutative torus and 3-sphere, with the hope of shedding some light on how (and if) Riemannian curvature fits into noncommutative geometry.

Thomas Weber (JMU Würzburg)

## Obstructions of Drinfeld Twist Deformation

**23. 10. 2015**, 14 Uhr c.t.

**Abstract**

One common topic in Hopf algebra technology is twist deformation. We recall the basic definitions and present how to deform a Hopf algebra via a Drinfeld twist. It turns out that furthermore every algebra which is a left module algebra for the undeformed Hopf algebra deforms to a left module algebra with respect to the deformed Hopf algebra. The deformed product can be written in terms of the twist and the module action. It’s easy to generalize these constructions to quantized enveloping algebras. So we receive a mighty tool to construct new examples of products on algebras out of given ones. A natural question is now if every product (or star product) can be induced in this way. Of course one would suggest “No!”, but in fact many examples of star products are of that form. Also this seems to be a non-trivial question since every Hopf algebra that acts on our algebra of interest could provide a twist that deforms a product of the algebra to the one we consider. There’s a really nice approach to exclude some quantized enveloping algebras. The obstruction for twists shifts to solutions of the classical Yang-Baxter equation in the underlying Lie algebra. This connection to Lie bialgebras is surprising but also desirable since there are many classifications and results due to e.g. Drinfeld and Etingof. We might probably only sketch this point of view or shift it to another talk. Unfortunately this only rules out some module structures and gives no final answer to our question. So we proceed in another direction. If the algebra is the space of smooth functions on a connected compact symplectic manifold and a star product on it induced by a twist of a quantized enveloping algebra there’s also a Lie algebra action on the manifold. We argue that this integrates to a Lie group action which is even transitive such that the manifold has the structure of a homogeneous space. Finally there’s Mostow’s characterization of compact homogeneous spaces by the Euler characteristic of the manifold. Having this in mind we can produce zillions of star products that can never be induced by a twist, e.g. any star product on a higher pretzel surface. So we have at least a countable infinity of counterexamples.

Matthias Schötz (JMU Würzburg)

## Duality of uniform spaces and bornological unital abelian partial $^*$-algebras

**16. 10. 2015**, 14 Uhr c.t.

**Abstract**

By the Gel’fand-Naimark theorem, the category of abelian $C^*$ -algebras is equivalent to the category of compact Hausdorff spaces via two functors $\Phi$ and $\mathcal{A}$ that assign to every abelian $C^*$-algebra $A$ the topological space $\Phi(A)$ of its characters with the weak-$^*$-topology and to every compact Hausdorff space $X$ the $C^*$-algebra $\mathcal{A}(X)$ of continuous compex-valued functions over $X$.

On the arrow-side, they simply map every continuous unital $^*$-homomorphism / every continuous function to its pull-back. This theorem lies at the heart of non-commutative geometry and strict deformation quantisation in the spirit of Rieffels work, where abelian $C^*$-algebras correspond to ordinary commutative geometry and classical observable algebras and non-abelian $C^*$ -algebras to non-commutative geometry and quantum observable algebras. If one tries to generalise this to the realm of e.g. Fréchet-$^*$-algebras, then one also has to generalise the Gel'fand-Naimark theorem: It is immediately clear that the construction of the space of characters of a general unital abelian $^*$-algebras and of the $^*$-algebra of complex-valued continuous functions over a general topological space still make sense, but they do not establish an equivalence of these two categories. Nevertheless, one can ask what the largest sub-categories of unital abelian -algebras and topological spaces are, such that this construction induces an equivalence of categories. This question was basically solved with the introduction of realcompact spaces by Edwin Hewitt. I suggest that this duality can be generalised even more by transfering the construction to uniform spaces and the partial $^*$-algebras of the uniformly continuous complex-valued functions over them. In this talk, I will give the definition of uniform spaces and (bornological) unital abelian partial $^*$-algebras, discuss their basic properties and explain the duality between these two categories.

# 17 Summer Term 2015

Yul Otani

## Deformation in quantum field theory

**10. 7. 2015**, 14 Uhr c.t.

**Abstract**

In this talk, I will talk about deformation on quantum field theory as a method for constructing models. After introducing the basic concepts of algebraic quantum field theory, I shall present Buchholz-Lechner-Summers' method of warped convolutions of Borchers triples. As an example, we shall see its use on the free scalar field.

Thorsten Reichert (JMU Würzburg)

## Classification of equivariant star products on symplectic manifolds

**26. 6. 2015**, 14 Uhr c.t.

**Abstract**

Building upon existing classification results for star products and invariant star products on symplectic manifolds $(M, \omega)$, we will discuss how to refine those to take quantum momentum maps into account. After introducing all relevant concepts, such as star products, equivalences and equivariant cohomology, we will sketch the construction of a bijection

$$\mathrm{Def}_{\mathfrak{g}}(M, \omega) \longrightarrow \mathrm{H}^2_{\mathfrak{g}}(M)[\![\nu]\!]$$

from the set of equivalence classes of equivariant star products on $M$ to it's second equivariant cohomology.

Bastian Seifert

## Takeuchi's Castle

**19. 6. 2015**, 14 Uhr c.t.

**Abstract**

Coalgebras are nice mathematical structures modelling the process of disassembling things. Important examples include the distributions, the divided power coalgebra and group-like coalgebras on sets. As for algebras it is, for physical reasons, important to have a notion of (co)representation equivalence, i.e. a Morita-like theory. In this talk we will give a short survey on coalgebras and comodules, then investigate the notion of Morita-Takeuchi equivalence of coalgebras. We will also examine the Picard group of a coalgebra, showing that it, unlike in the algebra case, always has a very nice structure.

Henrique Bursztyn (IMPA)

## Poisson homogeneous spaces and symplectic groupoids

**28. 5. 2015**, 14 Uhr c.t.

**Abstract**

Matthias Schötz (JMU Würzburg)

## Deformations for actions of Kähler Lie groups

**15. 5. 2015**, 14 Uhr c.t.

**Abstract**

I will give a summary on the geometric aspects of deformations for actions of Kähler Lie groups that have been discussed in great detail in a recent memoir by Pierre Bieliavsky and Victor Gayral. The idea is to deform a Fréchet-algebra $\mathscr{A}$ by making use of the action of some general negatively curved Kähler Lie group $G$ on $\mathscr{A}$, analogously to Rieffel's deformation of $\mathscr{C}^\infty(\mathbb{R}^{2n})$ using the action of $\mathbb{R}^{2n}$ via translation. As a concrete example, I will discuss the application to the Poincaré-disc.

Chiara Esposito (JMU Würzburg)

## Pre-preprint of the week: a funny reduction

**8. 5. 2015**, 15 Uhr c.t.

**Abstract**

In the present talk we present a new approach for reduction in the setting of Poisson Lie groups acting on Poisson manifolds, by using the techniques of algebraic and coisotropic reduction. Have fun!

Alexander Schenkel

## Gauge theories in locally covariant quantum field theory

**24. 4. 2015**, 14 Uhr c.t.

**Abstract**

Locally covariant quantum field theory (LCQFT) has been proposed by Brunetti, Fredenhagen and Verch as an axiomatic setting for describing quantum field theories on generic (globally hyperbolic) Lorentzian manifolds. In my talk I will give a brief introduction to LCQFT and present our results on the construction of (mostly Abelian) gauge theories in this framework. The most elegant and transparent construction of Abelian U(1)-gauge theory and its higher analogs given by connections on (higher) gerbes is obtained by making use of techniques from differential cohomology. The structure of the resulting quantum field theories can be analyzed in full detail and I will show that these theories violate some important axioms of LCQFT. Trying to understand and resolve the incompatibility between gauge theories and LCQFT naturally leads us to develop a homotopy theoretic generalization of LCQFT. I will present some first results in this direction, which in particular indicate that gauge field observables should be described by a homotopy cosheaf.

## Stefan Waldmann

# Geometric group theory and locally convex analysis: a marriage of heaven and hell

**17. 4. 2015**, 14 Uhr c.t.

**Abstract**

In this talk I will review some of the basic features of geometric group theory: finitely generated infinite groups carry a natural coarse geometry induced by the length function. For the algebraic group algebra one can define several locally convex topologies which give rise to various Fréchet group algebras. I will report on some of their properties. The results are joint work with Michel Cahen and Simone Gutt.

## Peter Bongaarts (Leiden Institute of Physics)

# Quantum Theory. A Mathematical Approach

**18. 3. 2015**, 15 Uhr c.t.

**Abstract**

In the past physics and mathematics were twin sciences. Newton ex- celled both in physics and mathematics. In the last half of the 19th and the first half of the 20th century mathematicians were deeply interested in new developments in physics and made important contributions. Think of Hilbert, von Neumann, Weyl, Cartan, van der Waerden. This is a thing of the past. Reasons? The Bourbaki movement which led to a more 'abstract' formulation of mathematics, which the physicists did not follow? Increasing specialization, coupled to publication pressure, which leads to short time research and prevents people from looking across the boundaries of their fields? The present generation of mathematicians does in general not know much of physics, in particular not much about recent developments. My claim is that the principles of the great new theories of 20th century physics, quantum theory and relativity, are easy to understand for mathematicians. The essential mathematical structure of these theories is simple. Mathematicians have sufficient familiarity with the advanced mathematics needed for this, differential geometry for relativity and functional analysis for quantum theory. Physicists lack this grasp, so they have to learn quantum mechanics, for instance, as a sequence of at first seemingly disconnected facts. Mathematicians who try to learn physics have a hard time; physics textbooks are inaccessible to them, because of the intuitive, heuristic, sloppy, and for them old fashioned mathematical language that is used. Mathematicians should be taught the principles of quantum theory in mathematical language. I did just that in a book that I published at the end of last year: "Quantum Theory. A Mathematical Approach". My lecture will be based on this book. This means that I shall give a short introduction to the main principles of quantum theory, in mathematical language.

Antonio de Nicola

## Sasakian nilmanifolds

**26. 2. 2015**, 14 Uhr c.t.

**Abstract**

Sullivan's theory of models can be used to get topological invariants of manifolds that are stronger than the de Rham cohomology ring. One defines a model for a manifold as a commutative differential graded algebra (CDGA) quasi- isomorphic to the algebra of differential forms. It is known that the minimal model of a compact nilmanifold is given by the Chevalley-Eilenberg complex of the corresponding Lie algebra. Recently, in his PhD thesis A. Tievsky constructed a finite-dimensional model of a compact Sasakian mani- fold. Comparing models for nilmanifolds and Sasakian manifolds, we give a classification of compact Sasakian nilmanifolds. This is joint work with B. Cappelletti-Montano, J. C. Marrero and I. Yudin

Giovanni Collini (Leipzig)

## Fedosov Quantization and Quantum Field Theory

**10. 2. 2015**, 14 Uhr c.t.

**Abstract**

In the context of deformation quantization for symplectic manifolds, a very geometrical construction has been proposed by Fedosov. Karabegov and Schlichenmaier developed a variant of his method in the case of almost-Kähler manifolds. The formal analogy between this deformation quantization and the perturbative approach to quantum field theory is discussed. In the $\phi^4$-interacting case, we present some results beyond the formal analogy.

Matthias Schötz

## Convergent star-products on Hilbert spaces

**31. 1. 2015**, 14 Uhr c.t.

**Abstract**

I will present a way to construct a Fréchet-topology on the symmetric tensor algebra over a Hilbert space, such that the usual star-products of exponential type are continuous. Due to the stronger assumptions than in previous approaches, one can proof some nice analytic properties of this deformation of the symmetric tensor product, e.g. a continuous dependence on a $C^*$-Algebra valued deformation parameter.

Stefan Waldmann

## Morita theory: an overview

**23. 1. 2015**, 14 Uhr c.t.

**Abstract**

In this talk I will give a short overview on Morita theory of rings and of *-algebras. The classification of Morita equivalent star products will be sketched.


## Paul Stapor

## A (rather) explicit formula for the Gutt-star-product

**9. 1. 2015**, 14 Uhr c.t.


**Abstract**

The Gutt-star-product is the deformed product on the symmetric tensor algebra $S^\bullet(\mathfrak{g})$ for a Lie- algebra g which comes from the Poincaré-Birkhoff-Witt-Theorem (or the Baker-Campbell-Hausdorff- series). We will to give a formula for a special case and try to get an idea, what this could look like in the general situation. The final aim of this attempt will be (one day), to get to know the number of terms which appear in this star product in order to get estimates on the growth.


## Chiara Esposito

## Rigidity of Hamiltonian actions I

**5. 12. 2014**, 14 Uhr c.t.


**Abstract**

In this talk we aim to prove the rigidity of Hamiltonian actions in the canonical and Poisson-Lie setting. Rigidity problems can always be viewed as problems about openness of orbits in appropriate settings. In particular, we will show that infinitesimal rigidity implies rigidity. The first part of this talk will concern rigidity results for the canonical momentum map and then we try to generalize these results to the Poisson-Lie setting.


## Niek de Kleijn (Kopenhagen)

## Symplectic group actions on formal deformation quantization

**28. 11. 2014**, 14 Uhr c.t.


**Abstract**

We will start by giving a brief introduction to and motivation for the notion of formal deformation quantization. In particular we will single out the Fedosov construction for formal deformation quantizations of symplectic manifolds and give some insights to its usefulness and generality. When we have established this framework we will use it to look at (discrete) group actions on the deformation which arise from symplectic actions on the underlying manifold. In particular we will try to make clear what problems one may encounter when trying to define and recognize (classify) such actions. After a break for coffee and cookies we will go into the mathematical details of our approach to solving these problems. In particular we will show that such group actions are classified (when they exist) by certain group cohomology sets and we will provide some evidence to the computability of these. If time permits we would like to go into some less worked out ideas about the underlying general approach and the problem with working these ideas out. At this point insights, objections and questions from the audience are more invited than usual.

## Benedikt Hurle

# Generalisations of the Hochschild-Kostant-Rosenberg-Theorem for constant rank maps II

**14.11.2014**, 14 Uhr c.t.

### Abstract

The well known HKR-theorem describes the Hochschild cohomology of the algebra of smooth functions on a manifold $C^\infty(M)$, in fact $\mathrm{HH}^\bullet(C^\infty(M)) \cong \mathfrak{X}^\bullet(M)$, where $\mathfrak{X}^\bullet(M)$ are the multivectorfields on $M$. The aim of this talk is to generalize this to the situation $N \xrightarrow{p} M$ for two manifolds $M$ and $N$ such that $p(N)$ is a submanifold of $M$, and compute here the Hochschild-cohomologies $\mathrm{HH}^\bullet(C^\infty(M), C^\infty(N))$ and $\mathrm{HH}^\bullet(C^\infty(M), \mathrm{DiffOp}(N))$, where $C^\infty(N)$ and $\mathrm{DiffOp}(N)$ are considered as $C^\infty(M)$ bimodule using the pullback of $p$. The main tool for proofing this is the Koszul complex of $C^\infty(M)$.

## Benedikt Hurle

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## Andreas Döring (Uni Erlangen)

# The Spectral Presheaf as the Spectrum of a Noncommutative Operator Algebra

**24. 10.,2014**, 14 Uhr c.t.

### Abstract

The spectral presheaf of a nonabelian von Neumann algebra or $C^*$-algebra was introduced as a generalised phase space for a quantum system in the so-called topos approach to quantum theory. Here, it will be shown that the spectral presheaf has many features of a spectrum of a noncommutative operator algebra (and that it can be defined for other classes of algebras as well). The main idea is that the spectrum of a nonabelian algebra may not be a set, but a presheaf or sheaf over the base category of abelian subalgebras. In general, the spectral presheaf has no points, i.e., no global sections. I will show that there is a contravariant functor from the category of unital $C^*$-algebras to a category of presheaves that contains the spectral presheaves, and that a $C^*$-algebra is determined up to Jordan $^*$-isomorphisms by its spectral presheaf in many cases. Moreover, time evolution of a quantum system can be described in terms of flows on the spectral presheaf, and commutators show up in a natural way. I will indicate how combining the Jordan and Lie algebra structures can lead to a full reconstruction of nonabelian $C^*$- or von Neumann algebra from its spectral presheaf.

Piotr Hajac

# Braided join comodule algebras of Galois objects

**17. 10. 2014**, 13 Uhr c.t.


### Abstract

We construct the join of noncommutative Galois objects (quantum torsors) over a Hopf algebra H. To ensure that the join algebra enjoys the natural (diagonal) coaction of H, we braid the tensor product of the Galois objects. Then we show that this coaction is principal. Our examples are built from the noncommutative torus with the natural free action of the classical torus, and arbitrary anti- Drinfeld doubles of finite-dimensional Hopf algebras. The former yields a noncommutative deformation of a non-trivial torus bundle, and the latter a finite quantum covering. (Based on joint work with L. Dabrowski, T. Hadfield and E. Wagner.)

# 19 Summer Term 2014

Chiara Esposito

## Symplectic groupoids, basic definitions II

**27. 6. 2014**, 10 Uhr c.t.

**Abstract**

In this talk we aim to give a basic introduction to symplectic groupoids. We start with the notion of Lie groupoid, show some easy examples and we construct its infinitesimal object, the Lie algebroid. This allows us to define symplectic groupoids, which are Lie groupoids endowed with a symplectic form.

Chiara Esposito

## Symplectic groupoids, basic definitions I

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**Abstract**

In this talk we aim to give a basic introduction to symplectic groupoids. We start with the notion of Lie groupoid, show some easy examples and we construct its infinitesimal object, the Lie algebroid. This allows us to define symplectic groupoids, which are Lie groupoids endowed with a symplectic form.

Stefan Waldmann

## Phase Space Reduction of Star Products, an Overview

**13. 6. 2014**, 10 Uhr c.t.

**Abstract**

In this talk I will give a short overview on some, by now classical, results on the phase space reduction of star product.

Matthias Schötz

## A Fréchet-Topology on the Weyl algebra over Hilbert spaces II

**30. 5. 2014**, 10 Uhr c.t.

**Abstract**

The Weyl algebra over a Hilbert space $\mathfrak{H}$ can be interpreted as a deformation of the symmetric algebra over $\mathfrak{H}$, where the usual symmetric tensor product $\vee$ is replaced by a non-commutative product $\star_b$, depending on a bilinear form $b$. In this talk, I will present a way to extend the inner product of H to its symmetric tensor algebra such that the product $\star_b$ becomes continuous in the locally convex topology created by the extension of all continuous inner products on $\mathfrak{H}$. It will turn out that under some additional requirements, this topology is the coarsest possible.

## Matthias Schötz

# A Fréchet-Topology on the Weyl algebra over Hilbert spaces II

**23. 5. 2014**, 10 Uhr c.t.

### Abstract


The Weyl algebra over a Hilbert space $\mathcal{H}$ can be interpreted as a deformation of the symmetric algebra over $\mathcal{H}$, where the usual symmetric tensor product $\vee$ is replaced by a non-commutative product $\star_b$, depending on a bilinear form $b$.

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## Stephane Korvers

# The deformation quantizations of the unit ball in the space of $n$ complex variables

**16. 5. 2014**, 10 Uhr c.t.

### Abstract


In the paper "The Deformation Quantizations of the Hyperbolic Plane" (see Comm. in math. phys., 289(2), 2009, pp. 529-559), Bieliavsky, Detournay and Spindel gave an explicit realization of the space of all both formal and non-formal deformation quantizations on the Poincaré disk in the complex plane. This construction relies on the evolution of a second order hyperbolic differential operator that emerges from a curvature contraction process on the Poincaré disk. Certain solutions of this evolution equation define convolution operators that intertwine the deformation theory at the contracted level with that of the Poincaré disk. This talk will be devoted to the study of a generalization of this construction in the case of the unit ball in $\mathbb{C}^n$. Firstly, we will study the geometry of this bounded symmetric domain. Then, we will explicitly construct a hierarchy of PDE's that is intimately related with the geometric structure of our space and from which we can describe the deformation theory of the unit ball in $\mathbb{C}^n$. Finally, we will discuss the general resolution of this problem and some perspectives. This is a joint work with Prof. Pierre Bieliavsky.


## Hassan Alishah

# Hamiltonian Evolutionary Games

**9. 5. 2014**, 10 Uhr c.t.

### Abstract


I will introduce a class of o.d.e's that generalizes for polymatrix games the replicator equation on symmetric and asymmetric games. I will also show that the phase space of these games is a Poisson stratified space. Then, I will characterize the Hamiltonian systems of these "evolutionary" polymatrix games. This extends known results for symmetric and asymmetric replicator systems.

## Matthias Schötz

# A Fréchet-Topology on the Weyl algebra over Hilbert spaces

**2. 5. 2014**, 10 Uhr c.t.

### Abstract

The Weyl algebra over a Hilbert space $\mathcal{H}$ can be interpreted as a deformation of the symmetric algebra over $\mathcal{H}$, where the usual symmetric tensor product $\vee$ is replaced by a non-commutative product $\star_b$, depending on a bilinear form $b$.

In this talk, I will present a way to extend the inner product of $\mathcal{H}$ to its symmetric tensor algebra such that the product $\star_b$ becomes continuous in the locally convex topology created by the extension of all continuous inner products on $\mathcal{H}$. It will turn out that under some additional requirements, this topology is the coarsest possible.

## Benedikt Hurle

# Deformation of fiber bundles

**25.04.2014**, 10 Uhr c.t.

### Abstract

In this talk, I will present the topic of my master thesis. It is about deformation quantization of fiber bundels $P \to M$, where $M$ has a given star product $\star_M$. The aim is to find necessary and sufficient conditions for the existence of a bimodule or an algebra structure on $C^\infty(P)$, which is compatible with $\star_M$. Also some examples coming from the theory of quantum groups will be studied.

Alan Lai

## Deformation of Connections

**20. 2. 2014**, 10 Uhr c.t.

**Abstract**

What do people mean when they integrate over a large space like the connection space? Inspired by the formal definition of a path integral, I attempt to give an interpretation on a measure of the connection space in loop quantum gravity literature. I will end with another way of understanding the connection space in such a way that one can strictly deformation quantise it.

Thorsten Reichert (JMU Würzburg)

## Dolgushev's formality theorem on smooth manifolds: Part 7

**23.1.2014**, 10 Uhr c.t.

**Abstract**

This series of talks will provide an in-depth presentation of Dolgushev's quasi-isomorphism

$$T_{poly}(M) \longrightarrow D_{poly}(M)$$

In the last session of this series we will finally be able to assemble all the parts from the previous talks in order to obtain the desired quasi-iso.

Thorsten Reichert (JMU Würzburg)

## Dolgushev's formality theorem on smooth manifolds: Part 6

**16.1.2014**, 10 Uhr c.t.

**Abstract**

This series of talks will provide an in-depth presentation of Dolgushev's quasi-isomorphism

$$T_{poly}(M) \longrightarrow D_{poly}(M)$$

In part 6 we will use the Fedosov construction in order to extend Kontsevich's quasi-isomorphism between polyvectorfields and polydifferentialoperators on $\mathbb{R}^d$ (and it's formal closure) to a quasi-isomorphism between polyvectorfields and polydifferentialoperators on any smooth manifold. For this we will briefly introduce some important techniques, such as the spectral sequence of a double complex and the twisting procedure of a $_\infty$-morphism by a Maurer-Cartan element.

Martin Bordemann

## An unabelian version of T.Voronov's construction of $L_\infty$ structures.

**9. 1. 2014**, 10 Uhr c.t.

**Abstract**

In 2005 T.Voronov gave a rather useful explicit construction of an $L_\infty$ structure on a graded vector space $V$ which is supposed to be an abelian subalgebra complementing a subalgebra $H$ in a graded Lie algebra $G$ which he extended to the ambient Lie algebra $G$. His technique gave rise to some $L_\infty$ constructions attached to coisotropic submanifolds and the simultaneous deformation of associative or Lie algebras and their morphisms (work of Y.Frégier et al.). We generalize his construction to an $L_\infty$ structure on the quotient $G/H$ (and the extension) without assuming that there is an abelian subalgebra complement to $H$ in $G$. The construction simplifies a bit to some 'graded dressing transformation' if there is a (non)abelian subalgebra complement. The main idea is the observation that the quotient $U(G)/(U(G)H)$ of the universal envelopping algebra $U(G)$ of $G$ is a cofree coalgebra on which $G$ acts from the left by coderivations. This quotient had recently been studied in the trivially graded case by Calaque, Caldararu and Tu: using their result we can show that the generalized Voronov $L_\infty$ structure is isomorphic just to a differential (no higher brackets) iff the (graded) Atiyah (or Nguyen-van Hai) class of the Lie algebra pair $(G, H)$ vanishes. We shall indicate how the generalization may help to the quantization problem of coisotropic submanifolds as modules.

Simone Gutt

## Dirac operators in a symplectic context

**12. 12. 2013**, 10 Uhr c.t.

**Abstract**

On a symplectic manifold $(M, \omega)$, using a $Mp^c$ structure (which always exist) and a $Mp^c$ connection, one can define a symplectic Dirac operator. The $Mp^c$ group is a central extension of the symplectic group analogous to the $Spin^c$ group in Riemannian geometry. We shall speak about invariant $Mp^c$ structures and lifts of some special subgroups of the symplectic group to the group $Mp^c$. Fixing a positive compatible structure $J$ and an adapted connection, we shall recall the definiton of the symplectic Dirac-Dolbeault operators and give properties concerning the dimension of their kernels. We shall compare the situation with the classical Dirac operators associated to the metric defined by $\omega$ and $J$.

Joint work with Michel Cahen, Laurent La Fuente-Gravy and John Rawnsley

Thorsten Reichert (JMU Würzburg)

## Dolgushev's formality theorem on smooth manifolds: Part 5

**5.12.2013**, 10 Uhr c.t.

**Abstract**

This series of talks will provide an in-depth presentation of Dolgushev's quasi-isomorphism

$$T_{poly}(M) \longrightarrow D_{poly}(M)$$

In part 5 we will continue to discuss the Fedosov-construction and related concepts in order to establish a quasi-isomorphisms

$$U_T : T_{poly}(M) \longrightarrow \Omega(M, T_{poly})$$

$$U_D : D_{poly}(M) \longrightarrow \Omega(M, D_{poly})$$

between polyvectorfields (polydifferential-operators) on a smooth manifold and the exterior forms on that manifold with values in the formal polyvectorfields (polydifferential-operators) on $\mathbb{R}^d$.

## Thorsten Reichert (JMU Würzburg)

# Dolgushev's formality theorem on smooth manifolds: Part 4

**28.11.2013**, 10 Uhr c.t.


### Abstract

This series of talks will provide an in-depth presentation of Dolgushev's quasi-isomorphism

$$T_{poly}(M) \longrightarrow D_{poly}(M)$$

In part 4 we will continue to discuss the Fedosov-construction and related concepts in order to establish a quasi-isomorphisms

$$U_T : T_{poly}(M) \longrightarrow \Omega(M, T_{poly})$$

$$U_D : D_{poly}(M) \longrightarrow \Omega(M, D_{poly})$$

between polyvectorfields (polydifferential-operators) on a smooth manifold and the exterior forms on that manifold with values in the formal polyvectorfields (polydifferential-operators) on $\mathbb{R}^d$.


## Gandalf Lechner

# KMS functionals and KMS states for algebras of deformed quantum fields

**21. 11. 2013**, 10 Uhr c.t.


### Abstract

In certain covariant models of quantum field theory on Moyal Minkowski space, one considers the unital $^*$-algebra A generated by quantum fields deformed by warped convolution, a deformation procedure generalizing Rieffel's deformation of $C^*$-algebras to deformations of modules, where the deformation parameter varies over a Lorentz orbit. In this talk, the thermal equilibrium states (KMS states) of A are investigated. It is shown that to each temperature, infinitely many different normalized KMS functionals exist, but only a single one is positive. This result can be obtained by analyzing the algebraic structure of $\mathcal{A}$ (in particular, certain ideals and subalgebras), whereas direct computations using $n$-point functions run into problems. Joint work with Jan Schlemmer.


## Thorsten Reichert (JMU Würzburg)

# Dolgushev's formality theorem on smooth manifolds: Part 3

**14.11.2013**, 10 Uhr c.t.


### Abstract

This series of talks will provide an in-depth presentation of Dolgushev's quasi-isomorphism

$$T_{poly}(M) \longrightarrow D_{poly}(M)$$

In part 3 we will continue to discuss the Fedosov-construction and related concepts in order to establish a quasi-isomorphisms

$$U_T : T_{poly}(M) \longrightarrow \Omega(M, T_{poly})$$

$$U_D : D_{poly}(M) \longrightarrow \Omega(M, D_{poly})$$

between polyvectorfields (polydifferential-operators) on a smooth manifold and the exterior forms on that manifold with values in the formal polyvectorfields (polydifferential-operators) on $\mathbb{R}^d$.

Thorsten Reichert (JMU Würzburg)

## Dolgushev's formality theorem on smooth manifolds: Part 2

**7.11.2013**, 10 Uhr c.t.

**Abstract**

This series of talks will provide an in-depth presentation of Dolgushev's quasi-isomorphism

$$T_{poly}(M) \longrightarrow D_{poly}(M)$$

In part 2 we will begin to introduce the Fedosov-construction and related concepts in order to establish quasi-isomorphisms

$$U_T : T_{poly}(M) \longrightarrow \Omega(M, T_{poly})$$

$$U_D : D_{poly}(M) \longrightarrow \Omega(M, D_{poly})$$

between polyvectorfields (polydifferential-operators) on a smooth manifold and the exterior forms on that manifold with values in the formal polyvectorfields (polydifferential-operators) on $\mathbb{R}^d$.

Thorsten Reichert (JMU Würzburg)

## Dolgushev's formality theorem on smooth manifolds: Overview

**31.10.2013**, 11 Uhr c.t.

**Abstract**

This talk will provide an overview on how Dolgushev's formality theorem extends Kontsevich's result for $\mathbb{R}^d$ to arbitrary smooth manifolds. We will (depending on the audience) include a brief introduction to the language of DGLA's and $L_\infty$-morphisms as well as a motivational section on why this theorem is of great importance in the context of deformation quantization. We will then proceed to sketch the construction of the quasi-isomorphism $T_{\mathrm{poly}}(M) \longrightarrow \mathcal{D}_{\mathrm{poly}}(M)$.

Melchior Grützmann

## Courant algebroids and $\mathbb{N}$-manifolds

**24. 10. 2013**, 10 Uhr c.t.

**Abstract**

We introduce the important notions of Courant algebroids and $\mathbb{N}$-graded manifolds. The former have become popular with the notion of generalized complex geometry. We will show how the latter can be used to define a cohomology for the former.