Objective: A problem of central significance in computer vision is to capture the motion of objects of interest in a sequence of images. The optical flow field of apparent velocities is determined here by an optimal control approach.

Applications: Information about the spatial arrangement of objects and the rate of change of this arrangement can be computed, e.g., for medical imaging, vision robotics, terrain mapping, and particle image velocimetry. A benchmark problem is to compute the optical flow field for a taxi sequence:

Required: velocity field $w$ along which constant brightness $I$ is convected, i.e., $\nabla I \cdot w + I_t = 0$. The ambiguity implies a selection process is needed to specify $w$.

Variational Principle: resolve ambiguity imposing minimal divergence, $|\nabla w| = \min$, and thereby avoid unnatural light sources.

Optimal Control Formulation: given images $Y_k : \mathbb{R}^N \supset \Omega \to \mathbb{R}$, determine the optical flow field $w : \Omega \times [0, T] \to \mathbb{R}^N$ and (without data differentiation) a regularized intensity field $I : \Omega \times [0, T] \to \mathbb{R}$ by minimizing:

$$J(w, I) = \int_0^T \int_\Omega \left[ \sum_k \delta(t - t_k)|I - Y_k|^2 + \varphi(|w_t|) + \psi(|\nabla w|) + \gamma|\nabla \cdot w|^2 \right] dxdt$$

subject to $\nabla I \cdot w + I_t = 0$, $\Omega \times [0, T]$.

Optimality System: Use second order TVD schemes to solve hyperbolic equations for $I$ and a Lagrange multiplier $p$ for a given $w$, and use a second order multigrid scheme to solve an elliptic system for $w$ for given $I$ and $p$. Both a regularized intensity field $I$ and an optical flow field $w$ are achieved at convergence.