

Corrigenda to “Nonstandard Analysis”

The author wants to thank all readers who sent typos and corrections.

A special thanks goes in memoriam Egor A. Alekhno who passed away beginning of 2017 in the age of only 37. He found the vast majority of misprints and inaccuracies of the following list.

Thanks also to Frederic Sommer who found some typos in the list itself. One mistake (the lack of the Archimedean property in the hypothesis of Theorem 1.2) was observed on math.stackexchange.com. Thanks also to Svend Erik Fjord who observed that in Theorem 12.52, one has to assume that y is internal. Thanks also to Gábor Törö.

(Latest corrigendum is from December 7, 2015).

- p. 3, l. 1: read “objects” instead of “obects”.
- p. 4, l. -20: It should be “the relation $a \leq b$ implies $a + c \leq b + c$ for all c and $ac \leq bc$ for all $c \geq 0_X$.”
- p. 5, l. -16: read “ \mathbb{Q}_X ” instead of “ \mathbb{Q} ”.
- p. 6, l. -12: It should be “of a totally ordered *Archimedean* field X ”
- p. 6, l. -9: “ X ” should be “ \overline{X} ”.
- p. 6, l. -5: add “(The Archimedean property is needed to show $A + (-A) = 0_X$.)”
- p. 7, l. 15: A dot is missing at the end of the line.
- p. 10, l. 4: read “ $n^{-1} < \varepsilon$ ” instead of “ n^{-1} ”.
- p. 16, l. -8: read “ \mathbb{R}^3 ” instead of “ \mathbb{R}^2 ”.
- p. 17, l. 10: read “ ${}^i x$ ” instead of “ ${}^i a$ ”.
- p. 20, l. -9: read “ $\text{cns } \mathcal{L}$ ” instead of “ \mathcal{L} ”.
- p. 21, l. -5: read “ \mathbb{R} ” instead of $I(\mathbb{R})$.
- p. 25, l. 3: “sentence” should be “transitively bounded sentence”.
- p. 25, l. -16: “sentence” should be “transitively bounded sentence”.
- p. 27, l. -12: At the end of the formula is missing “ $\wedge (\forall \underline{x} \in X_1 : \exists \underline{y} \in X_2 : (\underline{x}, \underline{y}) \in \varphi)$ ”.
- p. 28, l. 9: read “sentence” instead of “formula” (also in the subsequent lines).
- p. 29, l. -7: read “ \widehat{S} ” instead of “ \widehat{X} ”.
- p. 29, l. -4: “on B, A ” should be deleted.
- p. 31, l. 1: read “ $*\Phi$ ” instead of “ Φ ”.
- p. 32, l. 8: “ B ” should be “ B_1, \dots, B_n ”; subsequently, there should be analogously “ $*B_k$ ” and “ B_k ” instead of “ $*B$ ” and “ B ”.
- p. 32, l. 13: read “ $*B_1 \times \dots *B_n$ ” instead of “ $B_1 \times \dots *B_n$ ”.
- p. 32, l. -13: read “an entity” instead of “a standard entity”.
- p. 32, l. -11: insert “and \underline{x} is the only free variable”.
- p. 32, l. -7: delete “standard”.
- p. 33, l. 7: read “ B ” instead of “ $\text{rng } f$ ”.

- p. 33, l. -9: read “ $*f$ ” instead of “ f ”.
- p. 34, l. 16 and 18: read “ $*\beta$ ” instead of “ β ”.
- p. 35, l. 17: read “ $*S_0 \in *S_1$ ” instead of “ $*S_1 \in *S_1$ ”.
- p. 37, l. -13 and -12: read “ \underline{z} ” instead of “ z ”.
- p. 38, l. -5: read “ φ ” instead of “ Φ ”.
- p. 39, l. 10 and 13: read “ φ ” instead of “ A ”.
- p. 41, l. -9: read “ y ” instead of “ \underline{y} ”.
- p. 41, l. -7 til end of page: The remark is actually not needed.
- p. 42, l. 7: Omit the word “not”.
- p. 42, l. 8: read “the contrary” instead of “this”.
- p. 46, l. 14: read “and” instead of “or”.
- p. 50, l. 6–8: read “ I_0 ” instead of “ I' ” or “ I ”.
- p. 52, l. -9: Read “ $\underline{x} = \underline{y}$ ” instead of “ $x = y$ ”.
- p. 55, l. -2: read “ ${}^\sigma A \subsetneq *A$ ” instead of “ $*A \subsetneq {}^\sigma A$ ”.
- p. 56, l. 16: read “ $*\Phi$ ” instead of “ Φ ”.
- p. 56, l. 18: read “ $(f_1(j), \dots, f_n(j))$ ” instead of “ $(f_{x_1}(j), \dots, f_{x_n}(j))$ ”.
- p. 56, l. -15: read “1.2” instead of “1.1”.
- p. 59, l. -11: read “ \mathbb{R} ” instead of “ R ”.
- p. 60, l. -11: read “on \mathbb{R} and $\mathbb{R} \setminus \{0\}$, respectively” instead of “on \mathbb{R} ”.
- p. 60, l. -10: read “ $f_1: * \mathbb{R} \rightarrow * \mathbb{R}$ and $f_2: * \mathbb{R} \setminus \{0\} \rightarrow * \mathbb{R} \setminus \{0\}$ ” instead of “ $f_i: * \mathbb{R} \rightarrow * \mathbb{R}$ ”.
- p. 63, l. -3: read “ \mathbb{N} ” instead of “ N ”.
- p. 64, l. -8: read “ \mathbb{N} ” instead of “ N ”.
- p. 65, l. 4: read “ \mathbb{N} ” instead of “ N ”.
- p. 66, l. 8: insert “ $x, y \neq 0$ ”.
- p. 66, l. -6: read “ $(N + 1)$ ” instead of “ N ”.
- p. 66, l. -6: read “ \mathbb{R} ” instead of “ ${}^\sigma \mathbb{R}$ ”.
- p. 68, l. 13: read “ $*x$ ” instead of “ x ”.
- p. 68, l. 13: read “ $\notin \text{fin}(* \mathbb{R})$ ” instead of “ $\in \text{fin}(* \mathbb{R})$ ”.
- p. 70, l. 9: read “5.11” instead of “5.6”.
- p. 72, l. 13: read “ \supseteq ” instead of “ \subseteq ”.
- p. 74, l. 10: read “ \mathbb{N}_∞ ” instead of “ $* \mathbb{N}_\infty$ ”.
- p. 75, l. 12: read “ \mathbb{N}_∞ ” instead of “ $* \mathbb{N}_\infty$ ”.
- p. 75, l. 2 and 5: read “ ${}^\sigma \mathbb{R}_+$ ” instead of “ ${}^\sigma \mathbb{R}$ ”.
- p. 75, l. 10: read “ $* \mathbb{R}$ ” instead of “ ${}^\sigma \mathbb{R}$ ”.
- p. 75, l. -12: read “ \mathbb{N}_∞ ” instead of “ $* \mathbb{N}_\infty$ ”.
- p. 77, l. -1: read “ \geq ” instead of “ $=$ ”.
- p. 78, l. 4: the second “ $=$ ” should be “ \subseteq ”.
- p. 78, l. -11: omit “of”.
- p. 78, l. -3: read “bijectively onto” instead of onto.
- p. 78, l. -2: read “by Exercise 8” instead of “by Example 8”.

p. 78, l. -13: The proof should read: “There is an entity $A_0 \in \widehat{S}$ with $A \in {}^*A_0$. Let \mathcal{F} denote the family of all functions of subsets of A_0 to subsets of A_0 . Let $\alpha(\underline{x})$ denote the attribute

$$\exists \underline{y} \in \mathcal{F} : (\text{“}\underline{y} : \underline{x} \rightarrow \underline{x} \text{ is one-to-one”} \wedge \exists \underline{z} \in \underline{x} : \underline{z} \notin \text{rng } \underline{y}),$$

i.e. for every fixed $x \subseteq A_0$, the sentence $\alpha(x)$ is bounded and is true if and only if x is Dedekind finite. Similarly, let $\beta(\underline{x})$ denote the attribute

$$\exists \underline{y} \in A_0^{\mathbb{N}} : \exists \underline{n} \in \mathbb{N} : \text{“}\underline{y} : \{1, \dots, \underline{n}\} \text{ is bijective”}.$$

Then

$$\forall \underline{x} \in \mathcal{P}(A_0) : (\alpha(\underline{x}) \iff \beta(\underline{x}))$$

is a bounded true sentence. The $*$ -transform of this sentence yields the assertion in view of the characterization of ${}^*\mathcal{F}$ and ${}^*\mathcal{P}(A_0)$ by Exercise 83 and Theorem 3.21.”

p. 79, l. -20: read “ ${}^*\mathbb{N} \rightarrow {}^*U$ ” instead of “ ${}^*U \rightarrow {}^*\mathbb{N}$ ”.

p. 79, l. -13 and -7: read “ A ” instead of “ *A ”.

p. 80, l. 11: read “ $A \subseteq {}^*U$ (by Theorem A.4)” instead of “ ${}^*A \subseteq {}^*U$ (because $A \subseteq U$)”.

p. 80, l. -2 and -1: read $\{1, \dots, h\}$ instead of $C := \{1, \dots, h\}$ and put instead $C := \{1, \dots, k\}$.

p. 81, l. 21: The displayed formula should be

$$\forall \underline{x} \in \mathcal{F} : (\exists \underline{y} \in \mathcal{A} : \text{dom}(\underline{x}) \subseteq \underline{y}) \wedge (\exists \underline{z} \in \mathcal{C} : \text{rng}(\underline{x}) = \underline{z}) \implies (\exists \underline{z} \in \mathcal{B} : \text{rng}(\underline{x}) = \underline{z}).$$

p. 82, l. -5: delete “)”.

p. 82, l. -3: read “ \underline{z} ” instead of “ vz ”.

p. 82, l. 5: add “ $\underline{y} \in \underline{x} \wedge$ ” to the left of the formula.

p. 83, l. -7: read “ $\underline{z} \in \mathcal{F} \times {}^*\mathbb{N} \mid$ ” instead of “ $\underline{z} \mid$ ”.

p. 83, l. -2: read “ x ” instead of “ f ”.

p. 85, l. 6: read “ δ -incomplete” instead of “ δ -free”.

p. 85, l. -12: read “ ${}^*\mathbb{R}$ ” instead of “ \mathbb{R} ”.

p. 86, l. 15: read “ *x ” instead of “ x ”.

p. 87, l. 6: read “ $+\infty$ ” instead of “ ∞ ”.

p. 87, l. 9 and 11: read “ ${}^*\mathbb{N}$ ” instead of “ \mathbb{N} ”.

p. 87, l. 14: read “ \underline{n} ” instead of “ n ”.

p. 87, l. -10: read “ *x ” instead of “ x ”.

p. 87, l. -10, -5, and -3: read “ *x_h ” instead of “ x_h ”.

p. 87, l. -4: read “ x_n ” instead of “ x ”.

p. 88, l. 4: read “ *x_h ” instead of “ x_h ”.

p. 89, l. -6: read “ *x_h ” instead of “ x_h ”.

- p. 90, l. 9: read: “More precisely, A is unbounded from above if and only if *A contains an infinite positive element, and A is unbounded from below if and only if *A contains an infinite negative element.”
- p. 90, l. 17: read “ *A ” instead of “ A ”.
- p. 91, l. 6: read “ A ” instead of “ *A ”.
- p. 91, l. -2: read “transfer” instead of “permanence”.
- p. 92, l. 1: read “ x ” instead of “ *x ”.
- p. 92, l. 2: read “Lemma 3.5” instead of “the permanence principle”.
- p. 93, l. -6: read “ ${}^*[a, b]$ ” instead of “ $[a, b]$ ”.
- p. 94, l. -2: read “ *x_0 ” instead of “ x_0 ”.
- p. 95, l. 5: delete “ $dx := x - x_0$ resp.”.
- p. 95, l. 7: read “ *x_0 ” instead of “ x_0 ”.
- p. 95, l. 11: read “ $0 \neq dx$ ” instead of “ h ”.
- p. 95, l. 12: read “ *f ” instead of “ f ”.
- p. 96, l. 1: read “ ${}^*(g'(x_0))$ ” instead of “ $g'(x_0)$ ”.
- p. 96, l. 8: read “denominator” instead of “numerator”.
- p. 97, l. 3 and 16: read “ \forall ” instead of “ \exists ”.
- p. 97, l. 5: the first “ k ” should be “ n ”.
- p. 97, l. 8: read “ x_1 ” instead of “ x_0 ”.
- p. 97, l. -7: read “transfer” instead of “permanence”.
- p. 98, l. 4: read “ y_n ” instead of “ $y(\underline{n})$ ”.
- p. 98, l. 7: read “ $\varphi(x_{\underline{n}-1})$ ” instead of “ $\varphi(y_{\underline{n}})$ ”.
- p. 98, l. -11: read “ f ” instead of “ F ”.
- p. 98, l. -7: read “ $\sum_{n=0}^{h-1}$ ” instead of “ $\sum_{n=1}^h$ ”.
- p. 100, l. -12: read “ T_n ” instead of “ T_n^{-1} ”.
- p. 101, l. 15: read “ *g ” instead of “ g ”.
- p. 102, l. -10: The displayed formula should be

$$\|f(\cdot, x)\| = 1 + |\text{st}({}^*[2^h x] - 2[2^{h-1} x])| = 1 + \text{st}(|[2^h x] - 2[2^{h-1} x]|)$$

- p. 104, l. 7: read “ κ ” instead of “ \mathcal{A} ”.
- p. 104, l. -6: read “For a polysaturated map (and infinite S), we thus need that...”.
- p. 106, l. 9: read “ $\mathcal{A}_0 \subseteq \mathcal{P}(A_0)$ ” instead of “ $\mathcal{A}_0 \in \mathcal{P}(A_0)$ ”.
- p. 106, l. -8: read “ A ” instead of “ *A ”.
- p. 106, l. -1: read “ A_0 ” instead of “ A ”.
- p. 109, l. -7: read “Prove that” instead of “Then”.
- p. 109, l. -16: move “(axiom of choice!)” to line -22.
- p. 111, l. 4: read “ a ” instead of “ b ”.
- p. 111, l. 19: read “Prove that” instead of “Then”.
- p. 112, l. 3: read “concurrent” instead of “satisfied”
- p. 112, l. 4: Delete the sentence: “Assume that φ is satisfied on A .”

- p. 112, l. 12: read “satisfied on A ”.
- p. 112, l. -9: read “ $\mathcal{A} \in \widehat{S}$ ”.
- p. 112, l. -9: read “Theorem 8.8” instead of “Theorem 8.12”.
- p. 113, l. -3: delete “By the Loś/Luxemburg Theorem 4.14”.
- p. 113, l. -4: read “ $(F_0(j))(a)$ ” instead of “ $(F_0(j))(a)$ ”.
- p. 113, l. -4: read “ λ ” instead of “ J ”.
- p. 114, l. -14: delete “only”.
- p. 114, l. -12: read “ $A \in j \cap \mathcal{A}$ ” instead of “ $A \in j$ ”.
- p. 115, l. 8: read “we put $S_0 = S$ ”.
- p. 116, l. 6: read “four” instead of “six”.
- p. 117, l. 8: read “ \underline{x}_{j_1-1} ” instead of “ $\underline{x}_{j_1} - 1$ ”.
- p. 117, l. -3 to -1: read “ \mathcal{S}_ω ” instead of “ \mathcal{S} ”.
- p. 120, l. 8: read “ ω ” instead of “ ∞ ”.
- p. 120, l. -15: read “Theorem 9.10”.
- p. 121, l. 4: read “ $S_{n,k}$ ” instead of “ S_n ”.
- p. 121, l. -20 to -13: read “ m_φ ” instead of “ m_0 ”.
- p. 122, l. -9: delete “and $i_{\beta_1}^{\beta_2}$ ”.
- p. 123, l. -18: read “ β_{B_i} ” instead of “ β_B ”.
- p. 126, l. -5: read “ X_0 ” instead of “ X ”.
- p. 127, l. 12 and -16: read “linear extension of f ”.
- p. 127, l. -12: delete “ $\|y\| \|x\| \leq$ ”.
- p. 133, l. 3: in the sum read “ ξ_n ” instead of “ ξ_n ”.
- p. 133, l. 14: read “ $h_0 \leq h$ ” instead of “ $h_0 < h$ ”.
- p. 134, l. 4: delete “or by Theorem 7.1”.
- p. 134, l. -15: read “ c_h ” instead of “ c_n ”.
- p. 136, l. 1: read “Hahn-Banach” instead of “Banach-Mazur”.
- p. 136, l. 5 and 6: read “Hahn-Banach” instead of “Banach-Mazur”.
- p. 135, l. line -2 and -1: read: For all $n \in \{1, \dots, h\}$, we have $\eta_n \approx 0$, because

$$|\eta_n| = \left| \sum_{k=1}^n (\eta_k - \eta_{k-1}) \right| \leq \sum_{k=1}^h |\eta_k - \eta_{k-1}| \approx 0.$$

- p. 136, l. 5: The displayed formula should be after the equality sign

$$\begin{aligned} &= \left| \eta_1 \xi_1 + \eta_h \xi_{h+1} + \sum_{n=2}^h (\eta_n - \eta_{n-1}) \xi_n \right| \\ &\leq |\eta_1 \xi_1| + |\eta_h \xi_{h+1}| + \sum_{n=2}^h |\eta_n - \eta_{n-1}| (\|x\|_\infty) \approx 0. \end{aligned}$$

- p. 136, l. -2: read “ $h - h_0 \in \mathbb{N}_\infty$ ” instead of “ $h_0 \leq h$ ”.
- p. 138, l. -8: read “ $h - h_0 \in \mathbb{N}_\infty$ ” instead of “ $h_0 \leq h$ ”.
- p. 139, l. 17: read “ $\xi_n - \xi_{n+1}$ ” instead of “ $\xi_{n+1} - \xi_{n+1}$ ”.

- p. 140, l. -11: read “Theorem 10.6” instead of “Theorem 10.5”.
- p. 140, l. -3: read “ a_k ” instead of “ a_n ”.
- p. 142, l. -13 and -11: read “ $\beta(\underline{x}, \underline{y})$ ”.
- p. 143, l. 2: read “largest integer which is at most”
- p. 143, l. 4: delete “ $\underline{y} \in \mathcal{P}(\Sigma)$ ”, replace in the subsequent formula “ \underline{y} ” by “ \underline{x} ”, and delete “for the choice $y := \Sigma_0$ ”.
- p. 143, l. 8: read “ \mathbb{N} ” instead of “ N ”.
- p. 143, l. 9: read “ $^*\Sigma$ ” instead of “ Σ_0 ”.
- p. 144, l. -9: read “[0, 1)” instead of “[0, 1]”.
- p. 145, l. 11: read “ κ_n ” instead of “ κ_{n-1} ”.
- p. 145, l. 15: read “ A ” instead of “ A_0 ” for the whole line.
- p. 145, l. 8 to 15: for $j = 1, \dots, n - 1$ replace throughout x_j by y_j with $y_j \in \{x_j, 1 - x_j\}$. In line 11 read “ $\varepsilon_0 := \varepsilon / (\kappa_1 + \dots + \kappa_n)$ ”
- p. 145, l. -8: read “ $\varepsilon_0 := \varepsilon / k$ ” instead of “ $\varepsilon_0 := \varepsilon$ ”. For better understanding read “By our additional assumption about the form of the elements of A (and since there are no numbers as in the first case)...”.
- p. 146, l. -10: read “ μ_0 ” instead of “ μ ”.
- p. 146, l. -2 and subsequent 3 lines: read “ $B^* \oplus ^*y$ ” instead of “ $B \oplus y$ ”.
- p. 150, l. 14: read “ $U_1 \cap U_2$ ” instead of “ $U_1 \subseteq U_2$ ”.
- p. 151, l. 12: read “ $^*\mathcal{F}$ ” instead of “ \mathcal{F} ”.
- p. 152, l. -3 to -1: read “ $A \in$ ” and “ $\ni A$ ” instead of “ $A \subseteq$ ” and “ $\supseteq A$ ”.
- p. 153, l. -19: read “ F_β ” instead of “ F_α ”.
- p. 153, l. -2: read “ $> \max$ ” instead of “ $>$ ”.
- p. 155, l. -10: read “ $\mathcal{F} := \mathcal{U}(x)$ ”.
- p. 157, l. 8 and 11: read “ $2k(i - 1) + (j - 1)$ ” instead of “ $2ki + j$ ”.
- p. 157, l. -13: read “ \mathbb{R} ” instead of “ R ”.
- p. 157, l. 16: read “we have seen” instead of “we will see”.
- p. 157, l. -4: read “ $\text{ns}(^*X)$ ” instead of “ $\text{ns}(X)$ ”.
- p. 158, l. -16: read “transitively bounded internal predicate”.
- p. 159, l. 1: read “largest” instead of “smallest”.
- p. 159, l. -8: read “ U ” instead of “ *U ”.
- p. 160, l. 11: read “ $\text{st}^{-1}(x)$ ” instead of “ $\text{st}(x)$ ”.
- p. 165, l. -13: delete “(Corollary A.6)”.
- p. 165, l. -9: add “ y is internal and”
- p. 166, l. 1: read “ $^*x(^*i_k)$ ” instead of “ $^*x(i_k)$ ”.
- p. 166, l. 15: read “ *I ” and “ *i_0 ” instead of “ I ” and “ i_0 ”.
- p. 166, l. 19: read “ $\text{mon}(y)$ ” and “ $\text{mon}(z)$ ”.
- p. 168, l. 4: read “ $f(\mathcal{F}) \rightarrow f(x)$ ”.
- p. 168, l. -16: read “ $V \in \text{mon}(\mathcal{U}(f(x)))$ ”.
- p. 171, l. -15 and -13: read “ $V(x)$ ” and “ $V^{-1}(y)$ ” instead of “ $V^{-1}(x)$ ” and “ $V(y)$ ”.

- p. 173, l. -15 and 16: read " $V^2 \subseteq$ " instead of " $V^2 \in$ ".
- p. 174, l. -6: read " $F_{n_0} \times F_{n_0} \subseteq U$ ".
- p. 175, l. -9: delete "any $y \in {}^*X$ and".
- p. 175, l. -6: read " X " instead of " *X ".
- p. 176, l. 10: read " $V^2 \subseteq U$ " instead of " $V^2 \in \mathcal{U}$ ".
- p. 176, l. 17: read " \approx_\emptyset " instead of " \approx ".
- p. 177, l. -14: delete "(" and read " ${}^*U({}^*x_1)$ " and " ${}^*U({}^*x_n)$ ".
- p. 177, l. -13: read " $({}^*x_k, y)$ ".
- p. 178, l. 14: read " $U_1, \dots, U_n \in \mathcal{U}$ with ${}^*U_1 \cap \dots \cap {}^*U_n \subseteq$ ".
- p. 179, l. -9: read " \underline{x} ", " \underline{y} ", " \underline{z} " instead of " x ", " y ", " z ".
- p. 179, l. -1: read " *d_1 ".
- p. 180, l. 3: read " $<$ ".
- p. 180, l. 13: read " x_{D_N, n_N} " instead of " x_{n_N} ".
- p. 180, l. 14: read " $N = n_k$ ".
- p. 180, l. 16: read " $\mathcal{F} \rightarrow x$ ".
- p. 180, l. 19 to 21: read " $x_{D_0, 2n}$ " instead of " x_{2n} ".
- p. 181, l. 8: read " \tilde{X} " instead of " $\tilde{\mathcal{U}}$ ".
- p. 181, l. 18: read " \mathcal{U}_* " instead of " \mathcal{U} ".
- p. 181, l. 21: read " (x, y) " instead of " x, y " (three times).
- p. 182, l. -18: read " $x = [{}^*x] \in \tilde{V}([y])$ " instead of " $[y] \in \tilde{V}([{}^*x]) = \tilde{V}(x)$ ".
- p. 182, l. -14 and -13: read " $([y], [{}^*x])$ " and " $(y, {}^*x)$ ".
- p. 183, l. 8: read " \supseteq " instead of the last " $=$ ".
- p. 185, l. -7: read " $A(V) \subseteq U$ ".
- p. 187, l. 12: A space is missing.
- p. 189, l. -12 and -10: read " ${}^*\mathbb{R}$ " instead of " \mathbb{R} ".
- p. 190, l. -5: read " \mathcal{U}_* " instead of " \mathcal{U} ".
- p. 191, l. -9: read " X/U " instead of " X/O ".
- p. 192, l. 1 to 3: read " V " instead of " U ".
- p. 195, l. -8: read " \setminus " instead of " \neq ".
- p. 196, l. 3: read " \tilde{X} " instead of " \tilde{X} ".
- p. 197, l. 10: read " S_0 " instead of " Σ_0 ".
- p. 197, l. 12: read " Σ_0 " instead of " Σ ".
- p. 197, l. -6: read " $\sum_k \mu(A_{n,k})$ ".
- p. 198, l. 13 to 22: read " S_0 " instead of " S ".
- p. 200, l. 10 and 11: read " Σ_0 " instead of " Σ ".
- p. 200, l. 11: insert "pairwise different".
- p. 200, l. 24: read "internal *-finite sequence".
- p. 202, l. 11: read " D_{n_i} " instead of " D_n " and delete the final " $^{-1}$ ".
- p. 202, l. -9: read " μ_L " and " $D_n \in \Sigma$ ".
- p. 203, l. 13: read "internal mapping F ".
- p. 203, l. -3: delete one of the duplicate " $\mu_L(D_0) \leq$ ".

- p. 205, l. 12 and 17: read “ dx ” instead of “ ds ”.
- p. 206, l. 3: read “ dx ” instead of “ ds ”.
- p. 207, l. 13: read “ n ” instead of “ k ”.
- p. 214, l. 9: read “ I ” instead of “ $*I$ ” in the first union.
- p. 215, l. 6: read “ $*S_{n+2}$ ” instead of “ $*S_{n+1}$ ”.
- p. 218, l. -2: delete “standard”.
- p. 218, l. -1: read “Lemma 3.5” instead of “Corollary 3.11”.
- p. 219, l. 4: read “ x_n ” and “ S_k ” instead of “ x_k ” and “ S_n ”.
- p. 219, l. 7: read “ (x_1, \dots, x_n) ”.
- p. 220, l. 5 and 6: read “ $j_0 \in U$ ”.
- p. 220, l. -9: read “ F ” instead of “ M ”.
- p. 220, l. -2: read “Corollary 4.21” instead of “Proposition 4.19”.
- p. 221, l. 3: read “Corollary 4.21” instead of “Proposition 4.19”.
- p. 221, l. 5: read “ $= a_i$ ” instead of “ $= i$ ”.
- p. 221, l. 14: insert “and $J_n \notin \mathcal{U}$ for all n ”.
- p. 222, l. -15: read “ $<$ ”.
- p. 222, l. -14: read “ \mathbb{N}_∞ ” instead of “ $*\mathbb{N}_\infty$ ”.
- p. 222, l. -13: delete “ $\leq h$ ”.
- p. 222, l. -11 and -9: read “ $*\mathbb{R}$ ” instead of “ \mathbb{R} ”.
- p. 223, l. 5: insert “Define analogously \mathcal{G} with $\text{dom}(f) \subseteq V$ and $\text{rng}(f) \subseteq U$ ” and put in the subsequent displayed formulas “ $\underline{y} \in \mathcal{G}$ ” and “ $\underline{x}_2 \in \mathcal{G}$ ”, respectively.
- p. 223, l. -5: delete the two “ $*$ ”.
- p. 223, l. -4: read “ X ” instead of “ $*X$ ”.
- p. 224, l. 2: replace the second sentence by “(by Proposition 3.16)”.
- p. 225, l. 4: read “ $*(-1)$ ” instead of “ $*0$ ”.
- p. 225, l. 7: read “ $<$ ”.
- p. 227, l. 11: read “ $dg \approx *(g'(x_0)) dx \approx 0$ ”.
- p. 227, l. -7: delete “ $\text{st}(g(h, x)) =$ ”.
- p. 228, l. -8: read “ $*\mathbb{R}_+$ ” instead of “ \mathbb{R}_+ ”.
- p. 229, l. 1: exchange “sufficiency” and “necessity”.
- p. 229, l. 10 and 12: read “ A_0 ” instead of “ A ”.
- p. 229, l. -11: read “ $\leq p(x - x_1)$ ” instead of “ $\leq p(x + x_1)$ ”.
- p. 230, l. -7: read “internal $*$ -finite”.
- p. 231, l. -11: read “ $\#_{\mathbb{R}}(\cdot)$ ” instead of “ $\#_X(\cdot)$ ”.
- p. 233, l. -9 to -11: read “ ξ_{k+1} ” instead of “ ξ_k ” (four times).
- p. 234, l. 3: read “ \mathcal{F} ” instead of “ Fcl ”.
- p. 234, l. 7: read “ $\emptyset \notin \mathcal{F}_0$ ”.
- p. 234, l. 14: read “ $x \in A$ ” instead of “ $x \in A_0$ ”.
- p. 234, l. -11: read “ $a_0 \in X$ ” instead of “ $a_0 \in *X$ ”.
- p. 235, l. 4: read “ x ” instead of “ $*x$ ”.

- p. 235, l. 14 and 15: read “ $x \in X$ ” instead of “ $x \in {}^*X$ ”.
- p. 235, l. -5: read “ $<$ ” instead of “ $=$ ”.
- p. 236, l. 5: read “ \mathcal{U}_Y ” instead of the second “ \mathcal{U} ”.
- p. 236, l. 14: read “ $({}^*x, y)$ ”.
- p. 236, l. 19: read “ ${}^\sigma\mathcal{A}$ ” instead of “ \mathcal{A} ”.
- p. 236, l. 21: read “ $(U(x_1) \cup \dots \cup U(x_n))$ ”.
- p. 236, l. 23: read “ $(U(x_k))$ ” instead of “ ${}^*U(x_k)$ ”.
- p. 237, l. 3: read “ O ” instead of “ U ”.
- p. 237, l. 5 and 12: read “Cauchy principle for ${}^*\mathbb{R}$ ”.
- p. 238, l. 7: read “ O_0 ” instead of “ O ”.
- p. 238, l. 17: The solution should read: “The set $\Sigma = \text{dom}(\mu)$ is internal by Theorem 3.19. Let z denote a given internal * -finite sequence $A_1, \dots, A_h \in \Sigma$. There are entities $A, B, C \in \widehat{S}$ with $\Sigma \in {}^*A$, $\mu \in {}^*B$, and $z \in {}^*C$. Consider the sentence

$$\forall \underline{x} \in A, \underline{y} \in B, \underline{z} \in C : \alpha \implies \beta,$$

where α and β are shortcuts with the meaning

$$\text{“}\underline{x} \text{ is a set algebra”} \wedge \text{“}\underline{y} : \underline{x} \rightarrow [0, \infty] \text{ is an additive measure”} \wedge \text{rng}(\underline{z}) \subseteq \underline{x}$$

and

$$\bigcup \text{rng} \underline{z} \in \underline{x} \wedge (\text{“Range of } \underline{z} \text{ pairwise disjoint”} \implies \underline{y}(\bigcup \text{rng} \underline{z}) = \sum_{\underline{n}} \underline{y}(\underline{z}(\underline{n})))$$

respectively. The transfer of this sentence implies the statement for the choice $\underline{x} := \Sigma$, $\underline{y} := \mu$, $\underline{z} := z$.”

p. 239, l. 9: read “ $f(i)$ ”

p. 239, l. 10: read “ \bigcup ” instead of “ \bigcup^* ”.

p. 239, l. 10: read “ $A, B, C, A \times B \subseteq S_n$ ” instead of “ $C \subseteq S_n$ ”.

p. 245, l. 3: read “ *X ” instead of “ X ”.